A Computational Method on Analysis Dynamics of Tracking System

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Abstract

The condition for yielding resonance and anti-resonance and the admittance characteristics of MDOF free torsional vibration system are discussed in this paper. Using control theory and basic mechanics method, on the point of view of Electro-Mechanics integration, we have given a mathematical model of driving unit and load construction for an E—O theodolite. The results of a series computations with the model and practical experiments suggested this method can solve many actual engineering problems.

1. Introduction

The controlling system of the large size tracing-instrument. which is a combination of its optics, mechanics as well as electric. In order to meet the requirements of real usage, needs to have its controlling system has a satisfied stability, a rapid response and a good reliability in whole system while the instrument tracing an object. But yet it is not easy to make so. Because on one hand the tracking device itself trends toward multi-function and on the other hand more and more new technic, new material and new technology are adopted, as a result, the whole system is growing complexity. So it is being important to give a correct discription for structural load characteristics.

In view of vibration, the mass, stiffness and damping of an actua engineering structure distributes on the whole system continuously other than concentrates on seperated and ideal mechanical components respectively, so all structures are distributed-parameter system. But actually, dynamic characteristic of structure is investigated only in an interesting frequency range, it is available that dynamic behaviour of MDOF system is studied by means of lumped model.

In a specific frequency region, people who design structure can estimate dynamic characteristics of their construction and people who design control system can also estimate frequency response of their electric unit, but the former is lacking in control subsystem and the latter is lacking in structural load, thus, the global behaviour of the comprehensive system, especially property of the go-between of driving unit and load construction is very ambiguous. In this paper using control theory and basic mechanics method. we derived the dynamical model of driving device and the dynamical model of structural load respectively, then from the point of view Electro-Mechanics integration, treated the mechanical structure and driving device as a whole, we have given a mathematical model (i. e. combined transfer function) of driving unit and load construction for general real tracking system, this method not just simplified theoretical analysis calculation but proved conside with the experiment based on impedance test.

2. Admittance Characteristics of MDOF Free Torsional Vibration System

1. Motion description

In general, E—O tracking instrument is typical MDOF free torsional vibration system. The problem of torsional vibration in axle, which widely exists kinds of turning machinery systems. When we analyse characteristic and stability of a integrated engineering structural system. it has very important meaning to research the dynamic characteristics of axle's torsional vibration system.

The motion derivative equation of a N degree of freedom torsional vibration system can be expressed as

$$[J]\{\ddot{\theta}\} + [C_r]\{\dot{\theta}\} + [K_r]\{\theta\} = \{M(t)\} \qquad (1)$$

werhe $[J]$, $[C_r]$ and $[K_r]$ is inertia, stiffness and damping matrix respectively, and they all are N * N real symmetrical matrix, $\{M(t)\}$ is column matrix of external moment couple. In case of thought which mechanical impedance analysis is applied. eq. (1) can be simplified as

$$\{M\} = [Z]\{\theta\} \qquad (2)$$

or

$$\{\theta\} = [H]\{M\} \qquad (3)$$

here

$$[Z] = \begin{pmatrix} Z_{11} & Z_{12} & & & 0 \\ Z_{21} & Z_{22} & Z_{23} & & \\ & \ddots & \ddots & \ddots & \\ & & & & Z_{n-1,n} \\ 0 & & & Z_{n,n-1} & Z_{n,n} \end{pmatrix} \qquad (4)$$

and

$$[H] = [Z]^{-1} = \frac{\text{adj } [Z]}{\det [Z]} \qquad (5)$$

joint impedance matrix $[Z]$ is a diagonal matrix only included three lines.

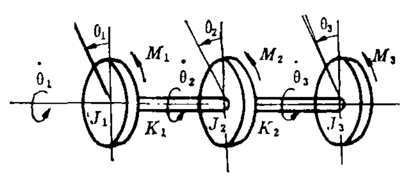

Fig. 1 3DOF torsional vibration system sketch

For fig. 1 shows 3DOF torsional vibration system. while ignor damping we can get

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \qquad (6)$$

and

$$\Delta(\omega^2) = \det[Z] = \begin{vmatrix} K_1 - \omega^2 J_1 & -K_1 & 0 \\ -K_1 & K_1 + K_2 - \omega^2 J_2 & -K_2 \\ 0 & -K_2 & K_2 - \omega^2 J_3 \end{vmatrix} \qquad (7)$$

2. Analysing resonance and anti-resonance

When damping is neglected. the displacement admittance matrix of a MDOF system is

$$[H_{ij}] = \frac{1}{\Delta} [\Delta_{ij}]' = \frac{1}{\Delta} [\Delta_{ji}] \qquad (8)$$

here, Δ: impedance matrix, that is $\det([k] - \omega^2[m])$

$[\Delta_{ij}]$: residual factor matrix

If a simple harmonic exciting force $F_j e^{j\omega t}$, is applied to point j of a system. the displacement response at point i of the system is

$$x_i = H_{ij} F_j = \frac{\Delta_{ji}}{\Delta} F_j \qquad (9)$$

As the exciting frequency ω is satisfied with

$$\Delta = |[k] - \omega^2 [m]| = 0 \qquad (10)$$

the resonance will take place at point i ($X_i \to \infty$); As the exciting frequency ω is satisfied with the eq. $\Delta_{ji} = 0$, the anti-resonance will occur at point i ($X_i \to 0$). Δ and Δ_{ji} are developed as real function of ω^2, it is high order algebra equation of ω^2

$$\Delta = a_0 (\omega^2)^n + a_1 (\omega^2)^{n-1} + \cdots + a_n = 0 \qquad (11)$$

$$\Delta_{ji} = b_0 (\omega^2)^m + b_1 (\omega^2)^{m-1} + \cdots + b_m = 0 \qquad (12)$$

here $m < n - 1$. thus, the number of resonant frequencies equal to n, ω^2_{Rs} (s = 1, 2, …n), and the number of anti-resonant frequencies equal to m. ω^2_{Ar} (r = 1, 2, …m). As long as the exciting frequency ω is equal to either ω_{RS} or ω_{Ar}, the resonance or anti-resonance will appear[1].

In researching for dynamic behaviour of engineering structure. resonance is well-known, anti-resonance is usually neglected. In fact, resonance and anti-resonance are both special converse vibration mode. For free system, the dynamic behaviour may be determined easy by studying its anti-resonance, especially, resonance and anti-resonance have special meaning in analysing a complicated system that exists mechanical and electrical coupling. Thus, In order to investigate the dynamical behaviour of a actural system, it is necessary to analyse the condition in which resonance and anti-resonance will take place.

3. Zero & Pole of Driving Device and Structure Load

In view of automatic control principle, eq. (11) is eigen equation of system. the solution of it is pole point of the system in complex domain, and solution of eq. (12) is zero point of the system. that is, pole is resonance point of system and zero is anti-resonance point of the system. Thus, through investigating the state of resonance and anti-resonance. such as its position and the number of them, a reliable basis to design a control system can be offered.

Driving device usually is DC servo motor with decelerated gear box or hydraulic motor with gear box. in recent years. torque motor is used to drive directly. In spite of the form of these drive equipment is different. their mathematics expression is the same. The transfer function of driving device is

$$M = K_m \cdot P_L \tag{13}$$

$$P_L = [1/(Y+CS)] \cdot (GX - K_m \cdot \theta_m) \tag{14}$$

here, M: torque of driving device

K_m: torque coefficient of driving device

P_L: differente potentical of driving device

Y: load bearing capacity coefficient of driving device

C: stiffness coefficient of driving device

G: gain of circuit

X: input

θ_m: torsional angle of motor

S: operator of Laplace

Its block-diagram is shown as fig.2

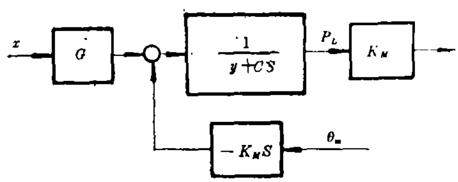

Fig. 2 Driving device block-diagram

For a system with direct torque motor, for the sake of its mass and elastic parts are comparatively discrete. it can be reduced into 3DOF system. if $\theta_m = \theta_2$, the external torbulence is neglected, $M_1 = M_3 = 0$, the combination velocity admittance is as follows

$$\frac{S\theta_2}{GX} = \frac{(1/K_m) \cdot [(J_1S^2/K_1+1) \cdot (J_3S^2/K_2+1)]}{\Delta'} \tag{15}$$

$$\begin{aligned}\Delta' = {} & (C/K_1K_2K_m^2)J_1J_2J_3S^6 + (Y/K_1K_2K_m^2)J_1J_2J_3S^5 \\ & + [(C/K_2K_m^2)(J_1+J_2)J_3 + (C/K_1K_m^2)(J_2+J_3)J_1 + (1/K_1K_3)J_1J_3]S^4 \\ & + [(Y/K_2K_m^2)(J_1+J_2)J_3 + (Y/K_1K_m^2)(J_2+J_3)J_1]S^3 \\ & + [(C/K_m^2)(J_1+J_2+J_3) + (J_1/K_1 + J_3/K_2)]S^2 \\ & + [(Y/K_m^2)(J_1+J_2+J_3)]S + 1\end{aligned} \tag{16}$$

thus, there are two anti-resonant peak in this system

$$\omega_{A1} = \sqrt{K_2/J_3} \tag{17}$$

$$\omega_{A2} = \sqrt{K_1/J_1} \tag{18}$$

It follows that the anti-resonant frequency of excitation point is only inherent characteristic of mechanical sub-structure, and the resonant frequency is not the solution of eq. (7), ω_{R1}, ω_{R2}, but it is the solution of eq. (16). $\Delta' = 0$. Now the characteristic parameter (Y, C etc.) of driving device join with mechanical structure parameter to form new ω_{R1}, ω_{R2}, the resonant frequency is the eigen value of the combination system included driving device[2], and

$$\omega_{A1} < \omega_{R1} < \omega_{A2} < \omega_{R2} \tag{19}$$

therefore, we can calculation zero and pole point of system usiong the eq. (15).

4. Computational Analysis and Test Result

In order to analyse dynamic characteristics of an E—O theodolite, which belongs to a turning system constituted with inner, external revolution pedestal and pitching axis mainly, it can be simplified into 3 DOF torsional vibration system. we made computation and syntheses on eq. (15) with practice parameters of all subsystems, dynamic characteristic curve is showed as fig. 3.

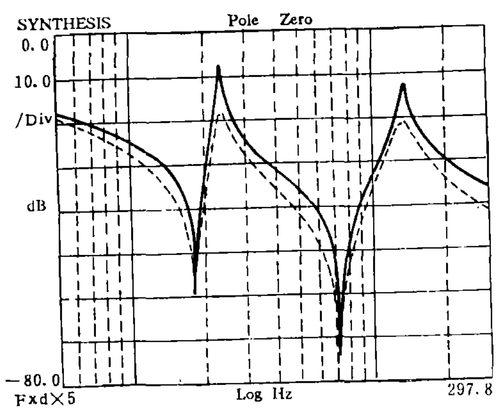

Fig. 3 Synthesis curve on computation

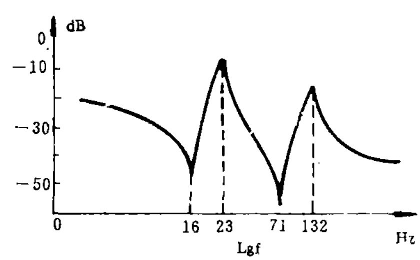

Fig. 4 Bode curve on test result

and in fig.4 that is a Bode curve measured, there is a better coincidence. The dash line in fig.3 is a synthesis curve that considering damping, we can see it borders further on fig.4 (in this paper the mathematical model included damping is neglected). Still further, according to the above mentioned model, by changed relevant parameters and made frequency synthesis. we found the effect of strengthening stiffness of sub-structures and increasing damping. a series of computations and simulations show that the above method can point out the direction of improving dynamics.

In recent, we analysed dynamics of a 3 axes servo revolution pedestal system in the same way. The system-level dynamics was given in quantitative analysis. The weakness of the integrated system was identified. We don't repeat the data in here, but surely it gave many important evidences for designing control system.

In this paper, the methematical models in 2DOF and 4DOF are not given, but it is available that simplified into 2DOF for a simple system, as Fig. 5 shows that is a electro-hydraulic servo system characteristic

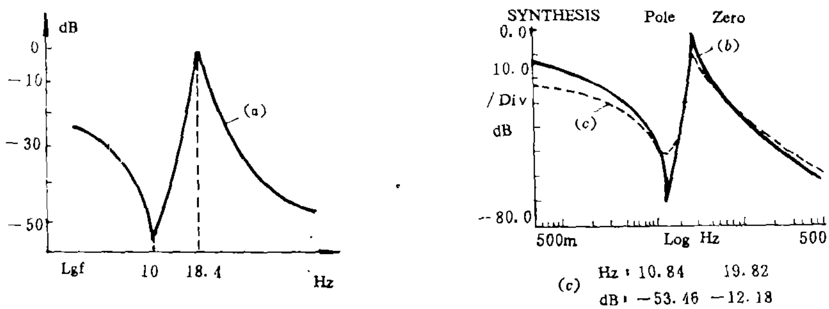

Fig. 5 Comparison of test and calcalation

curve. Of course, for a comparative complicated system or the interesting frequency range of greater width, the effect is better on 4DOF.

5. Conclusion

As prototype test shows that it is advisable to simplify the engineering structure into 3DOF or 2DOF system and hence to satisfy the requirement of project reality. It is much convenient to analyse a system by using impedance method, the identification of a project system can be carried out correctly.

Whenever the resonant and anti-resonant frequencies obtained, it is much easy to determine pole and zero points of the system. then the way for optimizing the dynamic characteristics through changing position distribution of zero and/or pole points and their numbers is available.

In set up mathematics model of a tracking system, it is important that the point of view Electro-Mechanics integration. Using the model in this paper, we can easy seek for sensitive parameters that influence system-level dynamics, so the model can provide evidence for designing electric control subsystem, can predict the dynamic behaviour of overall system for designer in the designing course, can also point out direction of imoroving dynamics and it is lead meaning for modelling in the finite element analysis too.

References

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跟踪系统的动特性分析的一种计算方法

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摘要：讨论了产生共振与反共振的条件及多自由度扭振系统的导纳特性。应用控制理论和基本力学方法，从机电一体化的角度给出了跟踪系统总体数学模型，计算与仿真结果表明应用这种方法可以较好地分析多自由度扭振系统的动力学特性。