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Influence of Finite Lower-level Lifetime on the Performance in CW Four-level Laser

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Abstract The rate equations for a continuous four-level laser are solved in this letter, taking into account the finite lower-level lifetime and the crystal field splitting of the laser levels. It is shown that the existence of this finite lower-level lifetime raises the threshold, nonlinearizes the in-and-out relationship and acts as an ultimate role in limiting the achievable output power for a four-level laser, instead of the ideal case of infinite achievable power while the relaxation from the lower-level is regarded as instantaneous.

Key Workds: Lower-lewel lifetime, Crystal field splitting, Boltzmann occupation factor

In the general analyses, the lower-level lifetime of a four-level laser scheme is often regarded as zero, i. e. the relaxation rate from this lower-level is assumed to be infinitely fast and thus the lower-level population is simply ignored[1-3]. This inevitably leads to the conclusion that with adequate pump power provided, output power of such systems can be scaled to any values. However, a finite lower-level lifetime does exist in four-level schemes. For example, the lifetime of the lower laser level $^4I_{11/2}$ for the 1.064 μm transition in the widely used crystal of Nd: YAG has been measured to range from 10 ns to 1 μs[4~7]. In the case of low power lasers, this finite lifetime does not reflect a significant influence on the laser performance. But when scaling to high powers, this lower-level population cannot be neglected and plays an important role in limiting the maximum achievable power. In this letter, we analyze the effect of a finite lower-level lifetime on the output power of four-level lasers, including proper accounting for the crystal field splitting. And we know that resulting from this finite lifetime, the output power cannot be increased infinitely.

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The energy level diagram of a typical four-level laser is shown in Fig. 1. Owing to the crystal field interaction, each level is split into several sublevels, as illustrated in the figure. In steady state pumping, the crystal field sublevels within each manifold are in quasi-thermal equilibrium at all times and thus their populations can be described by a Boltzmann distribution, as a result of the fact that relaxation between these sublevels can be considered instantaneous[8]. The laser transition occurs between two crystal field sublevels b and a with population densities of N_b and N_a within manifolds 2 and 1, respectively. The ratios of N_b and N_a to their respective total manifold population N_2 and N_1 are constants f_b and f_a, determined by the Boltzmann theory. Taking into account the degeneracies of the sublevels, the Boltzmann distribution factor f_a is given by

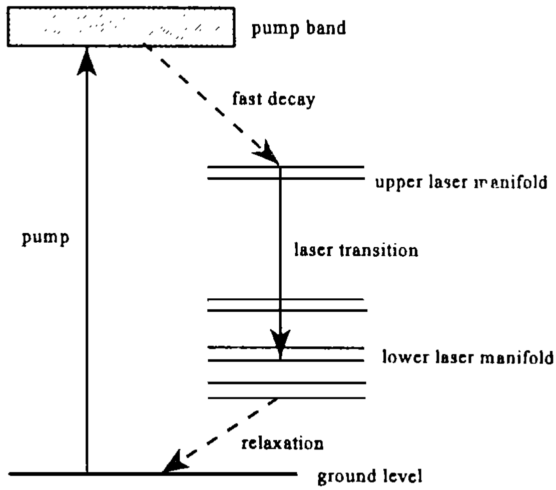

Fig. 1 Energy level scheme of a typical four-level laser crystal of Nd: YAG. The crystal field splittings within the laser manifolds are depicted.

$$f_a = \frac{g_a \exp(-E_a/kT)}{\sum_i g_i \exp(-E_i/kT)} \tag{1}$$

where the sum is over the crystal field sublevels in the lower manifold with the energies of E_i and degeneracies of g_i. A similar expression can be written for f_b.

We start from the four-level laser rate equations for a resonator of length L in which an active material of length l and refractive index n is inserted,

$$\frac{dN_2}{dt} = W_p N_g - c\sigma\varphi\left(f_b N_2 - f_a \frac{g_b}{g_a} N_1\right) - \frac{N_2}{\tau_2} \tag{2a}$$

$$\frac{dN_1}{dt} = c\sigma\varphi\left(f_b N_2 - f_a \frac{g_b}{g_a} N_1\right) + \frac{N_2}{\tau_2} - \frac{N_1}{\tau_1} \tag{2b}$$

$$N_g + N_1 + N_2 = N_t \tag{2c}$$

$$\frac{d\varphi}{dt} = \varphi\left[c\sigma\left(f_b N_2 - f_a \frac{g_b}{g_a} N_1\right)\frac{l}{L^*} - \frac{\delta}{t_r}\right] \tag{2d}$$

where $L^* = L + (n-1)l$, is the optical length of the resonator. W_p is pump rate, σ the stimulated emission cross section. τ_1 and τ_2 are the lifetimes of the lower and upper laser manifold, $t_r (= 2L^*/c)$ is the cavity round-trip time, δ is the fractional cavity loss per round-trip. In the above equations, we have assumed that no population exists in the pump band because of fast decay. The variants N_1, N_2, N_g are the population densities of the lower, upper manifolds, and the ground level. N_t is the total doping concentration of the crystal. φ refers to the photon density within the cavity. The term l/L^* in Eq. (2d) reflects the fact that even the light is amplified within the active medium of length l, the accretion of the photon density is some

l/L^* times lower.

Let $d\varphi/dt = 0$, we can have the expression for the population inversion at steady state from Eq. (2d),

$$\Delta N = f_b N_2 - (f_a g_b/g_a)N_1 = \delta/2\sigma l = \Delta N_{th} \tag{3}$$

which tells that though some small population lies in the lower laser manifold, the operational population inversion is still clamped on its value at threshold ΔN_{th}. This again verifies the widely-known fact that even above threshold, the enhancement in pump power leads to an increase in photon density rather than in the storage energy (population inversion).

Substituting Eq. (3) into Eq. (2a) and (2b), utilizing Eq. (2c), defining a relative lower-level occupation factor f as $f = (f_a/g_a)/(f_b/g_b)$, we can obtain the critical pump rate

$$W_{p,th} = \left[\frac{N_t}{\Delta N_{th}} f_b(\tau_2 - f\tau_1) - (\tau_1 + \tau_2)\right]^{-1} \tag{4}$$

where we may notice the effect of finite lower manifold lifetime τ_1 on the critical pump rate $W_{p,th}$. The longer this lifetime is, the higher the critical pump rate will be.

Let β be the amount by which threshold is exceeded, i. e. $\beta = W_p/W_{p,th}$, we can relate the introcavity photon density φ with β and the properties of the laser rod,

$$\varphi = \frac{\beta - 1}{c\sigma\tau_2 f_b[(1 + f)W_{p,th}\tau_1\beta + 1]} \tag{5}$$

In solid-state lasers, the cavity round-trip loss δ can be written as the sum of two terms,

$$\begin{aligned}\delta &= D + \ln R^{-1}\\ &= 2\alpha l + \ln(R'\prod_i T_i^2)^{-1} + \ln R^{-1}\end{aligned} \tag{6}$$

where R is the output mirror reflectivity and D the dissipative optical loss within the cavity describing the losses from R' (reflectivity of the rear mirror), T_i (the one way optical transmission of the ith internal element), and α (total loss coefficient in the laser rod).

If we write A_e as the equivalent cross sectional area of the laser medium occupied by the oscillating mode(s) (clearly, when ω_0 is the spot size at the center of the resonator, we have $A_e \approx \pi\omega_0^2/4$ if the laser is operating in TEM_{00} mode)[9]. Notice that the last term in Eq. (2d) represents the loss of the photon density within one round-trip time t_r and a portion of $\ln R^{-1}$ within the total loss δ is contributed as the useful output, so, from Eq. (5) and (6), we have the output power as

$$\begin{aligned}P_{out} &= h\nu V\varphi \ln R^{-1}/t_r\\ &= \frac{A_e I_s \ln R^{-1}}{2f_b} \cdot \frac{\beta - 1}{(1 + f)W_{p,th}\tau_1\beta + 1}\end{aligned} \tag{7}$$

where $h\nu$ is the photon energy of the laser transition and we have defined a gain saturation intensity I_s as $I_s = h\nu/\sigma\tau_2$.

From Eq. (4), (5), (7), we can find that the existence of a finite lower manifold lifetime does raise the critical pump rate and reduce the steady-state introcavity photon density and output power. A larger relative occupation factor f causes a greater influence to the lasing operation. As we have mentioned, Eq. (7) shows that the output power of a given four-level system cannot be scaled to infinite, even with adequate pump. In fact, when the pump power goes to infinite, i. e., $\beta \to \infty$, the output infinitely tends to its limit,

$$P_{out} \to P_{\infty} = \frac{A_e I_s \ln R^{-1}}{2f_b(1+f)W_{p,th}\tau_1} \tag{8}$$

Obviously, when $\tau_1 \to 0$, $P_{\infty} \to \infty$, which coincides the results of tradition treatments quite well. To make a comparison, we may find when we set $\tau_1 = 0$ and $f_b = 1$ (which is the case of the usual discussion), Eq. (4), (5), (7) go back to the familiar expressions in the text-book[1][2][9].

From Reference [9] and from the characteristic of $N_g \approx N_t$ in four-level lasers, we can relate pump rate W_p with the input pump power P_{in} as,

$$P_{in} = W_p V N_t h\nu / \eta_p \tag{9}$$

where η_p is the overall pump efficiency and V the laser rod volume. Substituting Eq. (4) into Eq. (9), we can get the threshold pump power P_{th}. As β is also the ratio of pump power to its critical value, $\beta = P_{in}/P_{th}$, we can obtain the important parameter of slope efficiency η_s as

$$\eta_s = dP_{out}/dP_{in} = \frac{A_e}{2P_{th}} I_s \ln\left(\frac{1}{R}\right)\left[(1+f)W_{p,th}\tau_1\beta + 1\right]^{-1} \tag{10}$$

which shows that for a given laser configuration, the slope efficiency is no longer a constant as in the ideal four-level systems when $\tau_1 = 0$. Instead, it behaves as a monotonously decreasing function of β. The highest efficiency occurs at the threshold when $P_{in} = P_{th}$.

As a conclusion, we have solved the rate equations for CW four-level lasers, taking the crystal field splittings and degeneracies of laser levels into consideration. When we account for the lower-level lifetime, we find that a nonlinear relationship appears between the input and output powers. The existence of this lower-level lifetime not only raises the threshold, but also causes an ultimately achievable limit for the output power, which is a point that must be allowed for when we scale the four-level lasers to high powers. From the above-derived equations, we may notice that the influence on the laser performance becomes more remarkable if materials with longer lower-level lifetimes are employed.

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有限的下能级寿命对四能级激光器工作特性的影响

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摘要　在考虑了有限的下能级寿命和激光能级与晶格场的相互作用的基础上, 对四能级激光的速率方程组进行了求解。结果表明由于这个有限寿命的存在, 激光阈值升高, 输出与输入之间出现了非线性。与下能级瞬时跃迁的理想情况不同, 泵浦功率增大时, 四能级激光器的输出趋向于一个极限值, 并且, 下能级寿命越长, 激光器所能获得的最大输出越小。

关键词: 下能级寿命; 晶格场分裂; Boltzmann 分布因子

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