

# Discussion on Temperature Profile in End-pumped Laser Rods

Yu Jin

(Changchun Institute of Optics and Fine Mechanics,  
Chinese Academy of Sciences, Changchun 130022)

**Abstract** In this paper, based on the analysis of the heating and cooling mechanism within an edge-cooled and end-pumped laser rod, we simplify the heat transfer equation in an axisymmetric system into a one-dimension one, taking into account only the radial heat flow. As an application, we solve the simplified equation to derive the expressions of the temperature distribution in two practical cases, that is, in laser rods end-pumped by a Gaussian beam or a top-hat beam, respectively. The results tell that a pump source with intensity of Gaussian shape causes to a higher temperature rise and a more severe nonuniform temperature distribution within the rod than that caused by a top-hat source with a pump radius the same as the former. The discussion is cast into a nondimensional form so that the results can be applied to various materials.

**Keywords:** End-pumping, Heat conduction, Nondimensionalize, Top-hat beam, Gaussian beam

## 1 Introduction

The problem of thermal deposition within a solid-state laser material under active pumping has been a key factor in limiting and degrading the laser performance since the first development of these lasers and has long been the subject of intense study. W. Koechner<sup>[1]</sup> discussed the temperature profile in an infinitely long rod pumped by uniformly distributed radiation and was seminal work in this area. But with the introduction of laser diodes (LDs) as pump sources and the employment of the technique of end-pumping, his work is obviously no longer valid. The nonuniform heating mechanism in end-pumping configuration leads to a much more complex temperature distribution within the laser rod and needs to be newly regarded. U. O. Farrukh et al.<sup>[2]</sup>, taking all the factors in an end-pumped rod into account, studied the transient process of thermal deposition and obtained complex expressions that bore little physical explanation. A. K. Cousins<sup>[3]</sup> and L. Yan et al.<sup>[4]</sup> limited their discussion to the laser rod end-pumped by a top-hat source, which is somewhat different from the output of a LD that can be well modeled by a Gaussian function.

In this paper, we present our analysis of the temperature profile in end-pumped laser rods. In the next section, we study the factors affecting the thermal conduction and simplify the end-pumping axisymmetric heat transfer equation into a one-dimension one by neglecting the axial heat flow, which is a reasonable approximation of the real situations. In the section that follows, we study the temperature distribution in laser rods axially heated by top-hat beam and Gaussian beam, respectively. The results show that a pump source with Gaussian profile leads to a more serious thermal program.

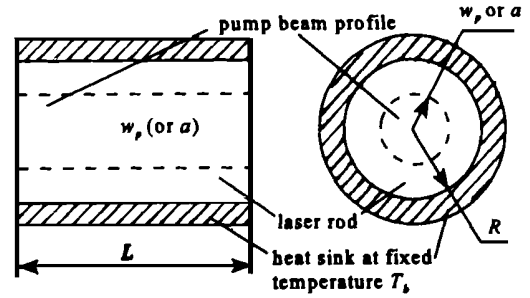


Fig 1 Side view and end view of an end-pumped rod with length  $L$  and radius  $R$ . Pump beam has a waist  $w_p$  or radius  $a$ .

## 2 Heat Conduction in End-pumped Rods

A typical end-pumped laser rod is shown in Fig. 1. As we know, each kind of solid-state laser must dissipate an appreciable amount of heat. In steady state, a laser medium operating in CW mode displays a temperature distribution governed by the heat transfer equation in axisymmetric system<sup>[5]</sup>,

$$k \cdot \nabla^2 T(r, z) = -q(r, z) \quad (1)$$

where  $k$  is the thermal conductivity of the laser material and  $q(r, z)$  represents the thermal power density of the heat radiation, that is, heat generated per unit volume per unit time. In all cases, we can describe the boundary conditions at any surface by the Newton's law of cooling,

$$k(T - T_\infty) + h \frac{\partial T}{\partial n} = 0 \quad (2)$$

In the above equation,  $h$  is heat transfer coefficient and  $n$  is the local normal to the boundary.  $T_\infty$  refers to the temperature evaluated at the ambient conditions. This condition means that at any surface, heat flux removed by the convection should equal the flux provided by conduction and can be extended to the cases of zero or constant boundary temperature ( $h \rightarrow \infty$ ), insulated boundaries ( $h = 0$ ) and convective heat transfer (finite  $h$ ).

To nondimensionalize the question and make it applicable to different materials, we express the thermal power density  $q(r, z)$  as the product of the total thermal power absorbed by the rod  $Q$  and a normalized scaling function  $f(r, z)$  that denotes the longitudinal and radial distribution of the heat source. Defining an aspect ratio  $A_r = L/(2R)$  and introducing the following variables,

$$r^* = r/R, z^* = z/L, T^* = \frac{T - T_\infty}{Q/(4\pi k L)} \quad (3)$$

we can rewrite the above heat transfer equation (1) and the boundary condition as,

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial T^*}{\partial r^*} \right) + \frac{\partial^2 T^*}{\partial z^{*2}} = -4\pi L R^2 \cdot f(r^*, z^*) \quad (4)$$

$$(B i) T^* + \frac{\partial T^*}{\partial n} = 0 \quad (5)$$

where  $B i$  is the Biot modulus defined as  $B i = hL/k$  that characterizes the relative magnitude of thermal conduction and convection.

Take a close look at Eq (4) and we may notice that three factors strongly influence the heat flow profile within a rod, the aspect ratio  $A_r$ , the absorption length  $\alpha$  (where  $\alpha$  is the absorption coefficient of the laser material at the pump wavelength), and the source pattern  $f(r^*, z^*)$ . When a discussion on the magnitude of order is performed, we find that both  $r^*$  and  $z^*$  are of order unity, so that the radial conduction term will be of order  $T^*$  and the axial conduction term of order  $T^*/A_r$ . Thus, for large aspect ratio rods, axial conduction will be negligible and the thermal transferring reduces to a one-dimensional problem. However, this does not imply that no axial gradient exists, as the source pattern  $f(r^*, z^*)$  shows a dependence on the axial position. The neglecting of axial flux merely means an emphasis on the fact that the temperature field is mainly determined by the source distribution and the radial conduction.

Now, let us turn our attention to the boundary condition Eq (5). Often, the thermal conductivity of the heat sink is much greater than that of the laser crystal. For example, the typical rod host material yttrium aluminum garnet (YAG) has a thermal conductivity of a factor of 30 smaller than that of the metal Cu, a common material for heat sink. As a result, a constant temperature boundary condition may be used as an excellent approximation on the cooled surface, in correspondence to an infinite Biot number here.

From the former discussion, we can get a conclusion that the heat transferring in an end-pumped rod can be simplified to a one-dimension problem with a fixed temperature at the cylindrical periphery. That is, the total problem can be simplified as

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial T^*}{\partial r^*} \right) = -4\pi L R^2 \cdot f(r^*, z^*) \quad (6)$$

$$T^* = 0, \quad r^* = 1 \quad (7)$$

where in Eq (7),  $T^*$  is now  $T_b$ .

### 3 Temperature Profile in Top-hat or Gaussian Beam Pumping

Usually the commonly employed pump radiation in the end-pumping scheme can be well modeled as top-hat (as the output of a fiber coupler) or circular Gaussian (as from a LD or a dye laser). We can easily apply Eq (6) to these cases through replacing  $f(r^*, z^*)$  with the appropriate functions

#### 1. top-hat pumping

In such case, pump power is uniformly distributed over the pumped region (say, that within a circle of radius  $a$ ) at the input plane of  $z = 0$ . Considering that the pump beam decays exponentially along the longitudinal direction due to the absorption by the laser medium and ignoring the change of pump-beam from the effects of thermal lensing, we can express the source function as

$$f(r^*, z^*) = \begin{cases} \alpha e^{-\alpha z^*} / [\pi a^{*2} R^2 (1 - e^{-\alpha})], & r^* \leq a^* \\ 0, & a^* < r^* \leq 1 \end{cases} \quad (8)$$

where  $a^* = a/R$  is the nondimensional radius of the pump beam.

Substituting Eq (8) into Eq (6), we have the solution to the problem as

$$T^* = \begin{cases} \frac{(\alpha L) \exp(-\alpha L z^*)}{1 - \exp(-\alpha L)} \left(1 - \frac{r^{*2}}{a^{*2}} - 2 \ln a^*\right) & r^* > a^* \\ -\frac{2(\alpha L) \exp(-\alpha L z^*)}{1 - \exp(-\alpha L)} \ln r^* & a < r^* < 1 \end{cases} \quad (9)$$

Clearly, for finite values of  $a^*$ , the temperature field is characterized by a parabolic profile within the pumped region and a logarithmic tail inside the surrounding ring area

2 Gaussian beam pumping

In the case of a laser rod end-pumped by another laser with a waist of  $w_p$ , the heat source can be represented by a Gaussian function. Thus, the normalized source profile is

$$f(r^*, z^*) = \frac{2\alpha}{\pi w_p^{*2} R^2 [1 - \exp(-\alpha L)]} \exp\left(-\frac{2r^{*2}}{w_p^{*2}} - \alpha L z^*\right) \quad (10)$$

accordingly,  $w_p^* = w_p/R$  is the nondimensional waist

Combining Eq (10) and Eq (6), we can express the nondimensional temperature field within the rod in the form of power series,

$$T^*(r^*, z^*) = \frac{(\alpha L) \exp(-\alpha L z^*)}{1 - \exp(-\alpha L)} \sum_{m=1}^{\infty} \frac{(-1)^m}{m \cdot m!} \left(\frac{2}{w_p^{*2}}\right)^m (r^{*2m} - 1) \quad (11)$$

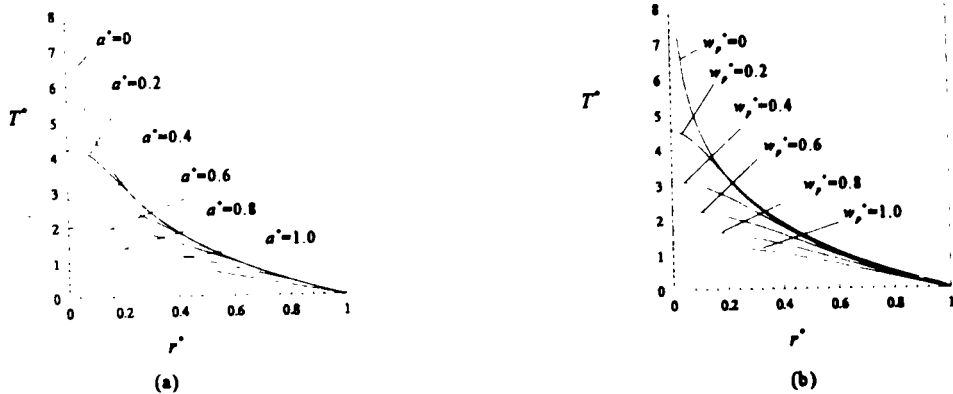


Fig. 2 The relationship between the nondimensional temperature  $T^*$  (in units of  $\alpha L \exp(-\alpha L z^*)/[1 - \exp(-\alpha L)]$ ) and the normalized radius  $r^*$ . (a) is for a top-hat pump source and (b) is for a Gaussian source

The results of Eq (9) and Eq (11) are both demonstrated in Fig 2(a) and (b) in which we have accounted for the fact that for an infinitesimal pump size, both cases should coincide with each other.

### 4 Discussion and Conclusion

On the whole, in this paper, we have studied the thermal conduction problem in an end-pumped laser rod and simplified it to a one-dimension problem. As an example of our theory, we computed the temperature distribution within the rod end-pumped by a top-hat beam and a Gaussian beam, respectively. From the analysis in the former sections and from Fig 2, we can draw the following conclusions:

1. An increase in pump power inevitably results in a large increment in the temperature

rise within the pumped rod. This is an obvious result when we remember that the total thermal power deposited within the rod  $Q$  is proportional to the incident pump power  $P_{in}$  and equals  $\eta_p P_{in} (1 - e^{-\alpha})$ , where  $\eta_p$  is the heat conversion coefficient including both fluorescent efficiency and quantum defect.

2 A small pump beam size leads to more intense temperature rise in the cases of both pump sources, which is easily seen from Fig. 2.

3 For the situation of the same pump size (that is,  $w_p^* = a^*$ ), the rod pumped by a Gaussian beam displays a more serious and more uneven temperature rise. As a matter of fact, when we examine Eq. (8) and Eq. (10), we may find that at the axial position of  $r^* = 0$ , the source function in Gaussian pumping case is as large as two times that in the top-hat case, and so this result seems more readily understandable.

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## 端面泵浦固体激光棒内温度分布的研究

余 锦

(中国科学院长春光学精密机械研究所, 长春130022)

**摘要** 在分析了端面泵浦、周边冷却的固体激光棒内生热及致冷机制的基础上, 将轴对称系统内的热传导问题简化为仅考虑径向热流的一维方程, 并推导出“大礼帽”型光束泵浦及高斯光束泵浦两种情况下激光棒内温度分布的表达式。结果表明, 在同样的光束尺寸下, 利用高斯光束泵浦将导致棒内更加剧烈和更加不均匀的温度变化。文中对讨论进行了无因次化处理, 以使所获得的结论能够适用于不同的激光材料。

**关键词:** 端面泵浦; 热传导; 无因次化; “大礼帽”光束; 高斯光束

**余 锦 男**, 1971年出生。1992年毕业于上海机械学院(现上海理工大学), 并获工学学士学位。现在中国科学院长春光机所攻读博士学位, 主要研究方向为半导体激光器泵浦的连续、脉冲固体激光器及其非线性频率转换。