第10卷 第4期
2002年8月

光学 精密工程
Optics and Precision Engineering

Vol.10 No.4
Aug. 2002

文章编号 1004-924X(2002)04-0333-07

Sphericity error evaluation using the genetic algorithm

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Abstract: To evaluate the form error of a sphere, a novel method based on the genetic algorithm was devised. In the paper, the problem about sphericity error is reviewed firstly. Secondly, the mathematical model of the sphericity error under the condition of minimum zone is derived from the mathematical definition of the circularity, and the fitness function of the algorithm is given as well. Thirdly, the critical such as selecting the initial value, initialing the population, and deciding genetic operators, etc. in the process of the algorithm implementation are described in detail. Finally, the method is verified by the examples in the literatures 1 and 2. The theoretical analysis and computation results indicate that the designed method not only conforms to the minimum zone condition but also assures a global solution, and is easily implemented. It is accurate and efficient in solving the problem of dimensional and tolerance evaluation.

Key words: sphericity error; fuzzy inference; genetic algorithm
CLC number: TH166 **Document code:** A

1 Introduction

The spherical surface is an important geometrical feature in the industry field. Because of the rapid improvement of high technologies, the demand for the machining precision of spherical surface becomes more and more strict, such as steam floating bearing. On the other hand to decrease the uncertainty of measurement results, the spherical surface used as datum feature must be more precise than the calibrated parts, for example the standard sphere on the Coordinate Measuring Machine (CMM). Therefore the inspection and evaluation of spherical parameters have been paid attention to.

Although there is no definition of sphericity in the dimensional and tolerance standard, much research work has been done in the aspects of tolerance criteria and assessment algorithms. Recently, the existing approaches are mainly based on the least squares sphere, minimum zone sphere, minimum circumscribed sphere and maximum inscribed sphere, among which the technique of least squares sphere is quite mature and accepted widely. But for other cases there have no efficient and uniform methods nowadays. The minimum zone sphericity error using the minimum potential energy theory was analyzed, which assumed an imaginary spring was located between the concentric spheres containing all sampling points[1]. The infinitesimal analysis method was used to analyze the characteristics of the sphericity function in the neighborhood of a local optimum solution[2]. A model_based approach to form tolerance evaluation using non_uniform rational B_splines was presented which must be under the conditions of computer aided design[3].

In the paper based on the genetic algorithm, a novel method of sphericity error evaluation is proposed. The genetic algorithm is used to search the optimum solution iteratively, and at the same time a fuzzy inference_based scheme is used to select the

Received date: 2002-01-14; **Revised date:** 2002-05-20
Foundation item: Key Project Supported by the National Natural Science Foundation of China(No. 59975025)

variables upper and lower bounds. In terms of the characteristics and advantages, the genetic algorithm shows its global search ability in the nonlinear optimization. It deals with codes of solution not solution itself; the search process of the algorithm starts with the population of solution not a single solution; it just uses the fitness function not the derivative or other auxiliary information; and it utilizes probable but not deterministic state transformation rules.

2 Mathematical model of sphericity error evaluation

According to the minimum zone condition, the sphericity zone can be defined as the minimum radial separation of the concentric spheres containing

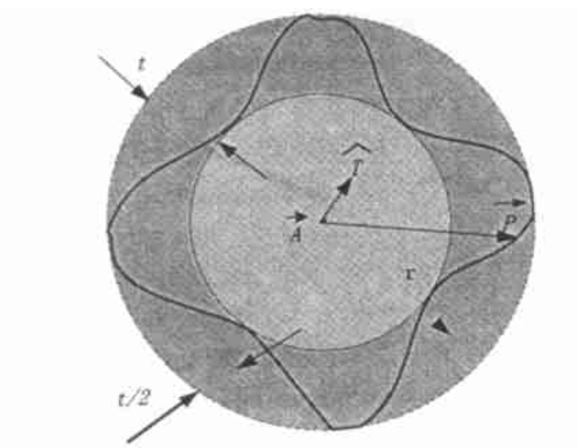

Fig. 1 Illustration of sphericity zone on the cross_section containing the sphere center.

all sampling points. The evaluation methods are based on the maximum inscribed sphere (MIS), minimum circumscribed sphere (MCS), least squares sphere (LSS) and minimum radial separation (MRS) spheres. In this paper, just the MRS is considered. Extended from the definition of circularity zone, the sphericity zone on the maximum cross_section of the sphere can be expressed by equations (1) (see Fig. 1)[1-4].

$$\begin{cases} \hat{\boldsymbol{T}}\cdot(\boldsymbol{P}-\boldsymbol{A})=0 \\ \left|\,|\boldsymbol{P}-\boldsymbol{A}|-\boldsymbol{r}\right| \leqslant \dfrac{\boldsymbol{t}}{2}, \end{cases} \quad (1)$$

Where: $\boldsymbol{A}$ is the sphere center; $\hat{T}$ is the radial direction from $\boldsymbol{A}$ to any sampling point; $\boldsymbol{P}$ is any sampling point; r is the distance from the sphere center to the middle of the tolerance zone; and t is the tolerance size.

Therefore, the sphericity evaluation under the condition of minimum zone is to search the center $\boldsymbol{A}$, which makes smallest the separation of the maximum distance and the minimum distance between $\boldsymbol{A}$ and any point.

$\boldsymbol{A}$ can be given as $\boldsymbol{A}\,(a+\triangle a_i(T),\ b+\triangle b_i(T),\ c+\triangle c_i(T))$, a, b and c are the initial values of the center coordinates and T is generation number. $\triangle a_i(T)$, $\triangle b_i(T)$ and $c_i(T)$ are the random variables between $(-k,\ k)$, where $(-k,\ k)$ are the given variables upper and lower bounds determined by the concrete problem (described later). Then the distance r_{ji} between the jth point and $\boldsymbol{A}$ $(a+\triangle a_i(T),\ b+\triangle b_i(T)\ ,\ c+\triangle c_i(T))$ can expressed as equation (2):

$$r_{ji}=\sqrt{\{x_j-[a+\Delta a_i(T)]\}^2+\{y_j-[b+\Delta b_i(T)]\}^2+\{z_j-[c+\Delta c_i(T)]\}^2} = r_{ji}(\overline{a_i},\overline{b_i},\overline{c_i}), \quad (2)$$

where: $\overline{a_i}=a+\Delta a_i(T)$, $\overline{b_i}=b+\Delta b_i(T)$, $\overline{c_i}=c+\Delta c_i(T)$. Then the objective function can be defined as equation (3) according to condition (1):

$$G_i(\Delta a_i(T),\ \Delta b_i(T),\ \Delta c_i(T)) = \min\{\max\}[r_{ji}(\overline{a_i},\overline{b_i},\overline{c_i})]-\min[r_{ji}(\overline{a_i},\overline{b_i},\overline{c_i})]\}, \quad (3)$$

So the fitness function is given by equation (4):

$$\text{fitness}(\Delta a_i(T),\ \Delta b_i(T),\ \Delta c_i(T)) = G_i(\Delta a_i(T),\ \Delta b_i(T),\ \Delta c_i(T))^{-1}, \quad (4)$$

If the algorithm ends at the generation T_0 and obtains the i_0th variable $(\triangle a^{i_0}(T_0),\ \triangle b^{i_0}(T_0)\ ,\ \triangle c^{i_0}(T_0))$, the accuracy is up to demand, then $\boldsymbol{A}$ and r can be given as follows:

$$\begin{cases} \boldsymbol{A}=(a+\triangle a^{i_0}(T_0),\ b+\triangle b^{i_0}(T_0),\ c+\triangle c^{i_0}(T_0)) \\ r=\dfrac{r_{\max}+r_{\min}}{2} \end{cases}, \quad (5)$$

Where: r_{max} and r_{min} are the maximum radius and minimum radius at the $i_0 th$ variable of the $T_0 th$ generation.

3 Genetic algorithm and its implementation techniques

The genetic algorithm is an adaptive and probabilistic search algorithm with an iterative procedure that simulates the evolution process of the nature. It generally consists of the following steps[5-6]:

(1) Initialing the population: select m individuals in the variables' range as the initial population.

(2) Selecting superior individuals for next generation: according to selection probability and selection schemes, randomly select m chromosomes, and reject the inferior ones.

(3) Crossover operation: in terms of crossover probability, crossover times and strategy, takes two individuals and produce two new individuals

(4) Mutation operation: the genetic location is replaced with a new random value according to the crossover probability, crossover times and scheme.

(5) Evaluating the population: in terms of the fitness function compute the fitness degree of every individual and determine the best chromosome. If the chromosome is the desired solution, end the evolution; continue steps of (2)_(4) called a generation until the solution appears. The evolution process is shown in figure 2. $P(t)$ is the population of the tth generation.

The Implementation techniques of the genetic algorithm in the problem are given as follows:

(1) Selecting the initial variable

The least squares solution is used as the initial value of the sphere center (a, b, c) because it is proved to approximate the ideal solution so that the proposed algorithm can converge rapidly.

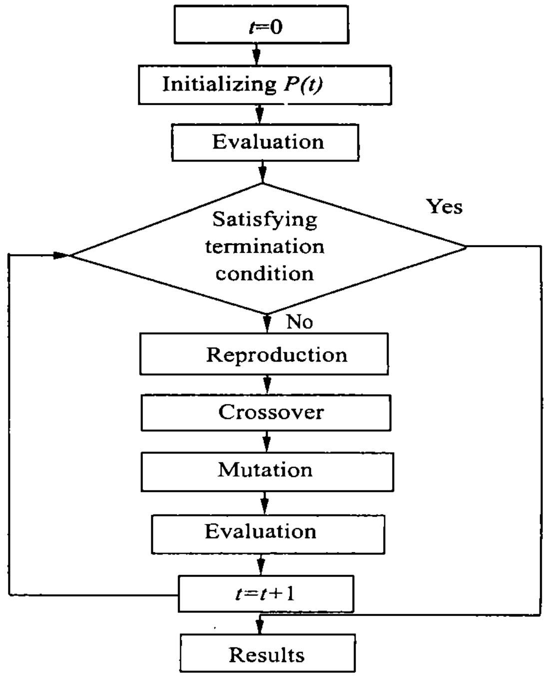

Fig. 2 Implementation process of genetic algorithm.

(2) Representing the chromosomes

The original representation way is binary. For the purpose of high accuracy and efficiency, the real number is used to express the chromosomes, which conforms to the practical parameters, namely one gene is corresponding to a real number. Michalewicz has proved that more natural representations are more efficient and produce better solutions[7-11]. Let $v_i = [\Delta a_i, \Delta b_i, \Delta c_i]$, v_i is the ith chromosome (variable) which is composed of three genes, namely Δa_i, , Δb_i and Δc_i.

(3) Initialing the population

Let the population size = POP_SIZE. Randomly select POP_SIZE groups of variables within $(-k, k)$, namely, $i = [\Delta a_i, \Delta b_i, \Delta c_i]$, $i = 1, 2, \ldots$, POP_SIZE.

In order to get the optimum solution rapidly and to assure the parallel search successfully, the variables' initial upper and lower bounds can be given by the fuzzy inference. Firstly according to least squares solution select a rough value. Then reselect the value in terms of the inference rules and the evaluation result. Since the initial error is bigger, a bigger bound value should be given. But with the solution approaching the accurate value, the bound should be smaller, see figure 3. Let C_1 be the least squares sphere and O_1 its center, and let $C_i(i = 2, 3, \ldots)$ be the center after the *ith* genetic iteration and O_i its center. With the generation number increasing, $e_i \rightarrow 0$, $O_i \rightarrow O_{mzc}$ (O_{mzc} is the center of the minimum zone sphere, namely the searched solution)

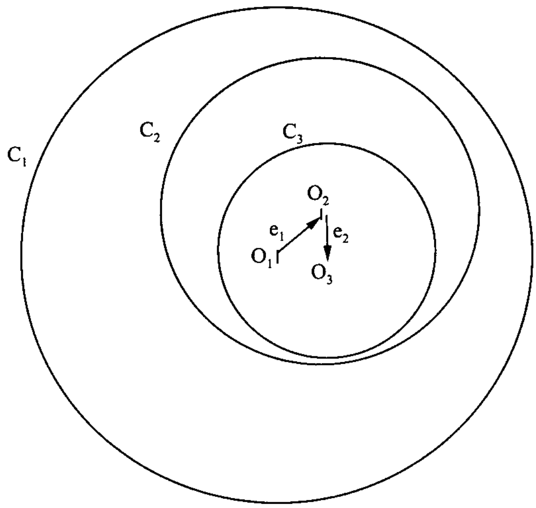

Fig. 3 Selection of the variables' upper and lower bounds.

Let k^β (β is a, b or c) be the size of variables' upper and lower bounds, the fuzzy inference rules are as follows:

If e_i bigger, then k^β bigger;

If e_i little bigger, then k^β little bigger;

If e_i smaller, then k^β smaller.

(4) Evaluation the population

According to the fitness function (4), compute the fitness degree of every chromosome and check whether the best one is satisfied or not.

(5) Genetic operations: selection, crossover and mutation

Selection: The roulette wheel based method developed by Holland is the first method. Here still use it to select the superior individuals. The probability P_i for each individual is defined by:

$$P[\text{ individual } i \text{ is chosen}] = \frac{F_i}{\sum_{j=1}^{POP\_SIZE} F_j}, \quad (6)$$

where F_i equals the fitness of individual i. According to the selection probability, the individuals with bigger values will be selected to make up of the new population.

Crossover: The simple arithmetic crossover is used to generate two new individuals for the population. It produces new chromosomes by two complementary linear combinations of the parent chromosomes. Firstly randomly produce a uniformly distributed number r between (0, 1). If $r < p_c$ (p_c is the crossover probability given by experience and experiment), randomly select two chromosomes v_i and v_j ($i, j = 1, 2, \ldots$, POP_SIZE) as the parent individuals for crossover operation, then the produced chromosomes v_i' and v_j' can be expressed as:

$$\begin{cases} v_i' = rv_i + (1-r)v_j \\ v_j' = (1-r)v_i + rv_j, \end{cases} \quad (7)$$

Mutation: The mutation strategy is to randomly select a mutation gene and alter it with a new random number. Through mutation a new individual can be obtained in one operation. Firstly randomly select one gene in terms of the mutation probability. Then replace it with a new value generated in the bounds of the variables randomly.

Through the above genetic operations from generation to generation, the genetic process will be terminated by some termination criteria when the accuracy of the solution is up to need.

4 Examples for verification of the algorithm

The verification examples are quoted from literatures 1 and 2. The computation conditions are shown in table 1 and the results of genetic iteration in table 2. The experiment results indicate that the solution of example 1 is 0.558μm smaller than the least squares solution, whose accuracy is increased by 7.04%. And the solution of example 2 is 0.34101mm smaller than the least squares solution, whose accuracy is increased by 10.18%. Figures 4 and 5 illustrate the relationship between sphericity error and iteration times. They also indicate that the algorithm rapidly converge to the accurate solution with the generation increasing.

Table 1　Computation conditions

Computation data	Initial sphere center	Population size	Mutation probability	Crossover probability
Group 1[a]	(0. 0, 0. 0, 0. 0)	30	0. 08	0. 35
Group 2[b]	(_0. 2796, _0. 6585, _0. 0006)	30	0. 06	0. 15

a. Simulation data in literature 1 as shown by appendix 1;

b. Simulation data in literature 2 as shown by appendix 2.

Table 2　Computation results

Computation data	Iteration sphere center	Iteration radius	Iteration times	Iteration error	Least squares solution
Group 1[a]	(0. 000993, 0. 000024, 0. 000058)	1. 005446	287	7. 928μm	8. 486μm
Group 2[b]	(− 0. 412356, − 0. 335014, _0. 326140)	15. 312686	366	3. 351375mm	3. 692380mm

a. Simulation data in literature 1 as shown by appendix 1;

b. Simulation data in literature 2 as shown by appendix 2.

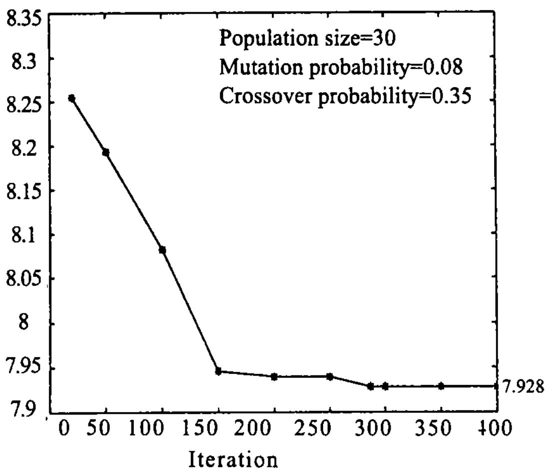

Fig. 4　Relationship between sphericity error and iteration times in literature 1.

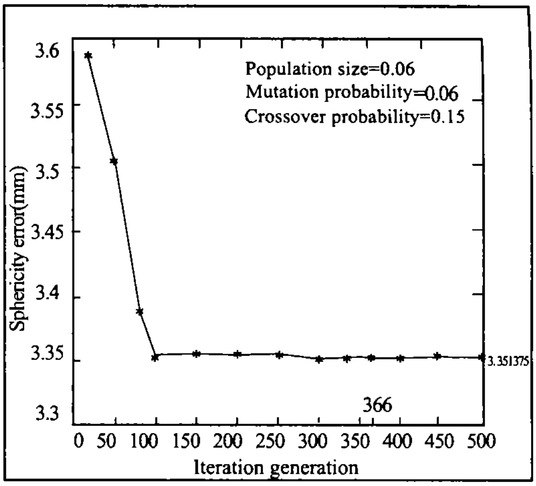

Fig. 5　Relationship between sphericity error and iteration times in literature 2.

5　Conclusion

Genetic algorithm simulates the natural process of evolution based on the strategy of " fittest for survival". The better individual (solution) can be obtained after such genetic operations as crossover and mutation, etc. It is an effective method to solve the nonlinear problem of the form error evaluation. Through parallel search and iterations, it can assure that the solution is the global one in the solution space[12]. The fuzzy inference simulates human intelligence to make decision, which can help the algorithm converge to the solution rapidly. The examples show that the algorithm is efficient, and easy to interpret and can give an accurate solution.

Appendix 1. Simulation data quoted in literature 1

No.	x	y	z	No.	x	y	z
1	0.81814	-0.43615	-0.38525	26	0.36179	-0.07902	0.93410
2	-0.80885	-0.53045	0.27905	27	-0.78624	0.20326	-0.59063
3	0.03749	0.90643	-0.42737	28	-0.46529	0.69616	-0.55167
4	0.86935	-0.27403	-0.42518	29	-0.93402	-0.02267	-0.35813
5	-0.50786	-0.07441	0.86060	30	0.43016	0.49319	-0.76122
6	0.75348	-0.67012	-0.00610	31	-0.46929	0.11501	0.88119
7	0.05867	-1.00773	-0.00198	32	0.60788	-0.41130	0.68580
8	-0.17486	0.60442	0.78297	33	0.70712	0.46598	0.53998
9	0.71121	-0.50123	-0.50042	34	0.67398	-0.38068	-0.63956
10	-0.47493	-0.51466	0.71992	35	-0.06245	-0.50544	0.86696
11	0.06016	-0.74129	-0.67432	36	-0.09577	0.96216	0.26433
12	0.05337	0.02006	1.00515	37	0.29318	0.79379	0.54237
13	0.29869	0.87920	0.38988	38	-0.39873	-0.68009	0.61867
14	-0.15488	-0.86697	0.48585	39	-0.30833	-0.07805	0.94937
15	0.38994	-0.00060	-0.92592	40	-0.70424	-0.64359	0.31476
16	0.00776	0.73922	0.68741	41	0.67010	-0.17187	-0.72841
17	0.62090	0.52006	-0.59341	42	-0.12359	-0.70688	0.70559
18	0.00952	-0.03493	-1.00376	43	-0.09694	0.64820	-0.75729
19	-0.44592	0.76584	-0.47827	44	-0.13014	0.04876	0.99815
20	0.01069	0.99511	0.11265	45	-0.28485	0.86262	-0.43728
21	-0.93371	-0.17351	-0.33160	46	0.16745	-0.11142	-0.98603
22	-0.48121	-0.69836	0.53717	47	0.00946	-0.00746	1.00873
23	0.50394	0.86933	0.03322	48	0.15374	-0.33634	-0.93141
24	0.80924	0.07332	-0.59398	49	-0.44828	0.85926	-0.25853
25	0.84569	0.06876	-0.53570	50	-0.61673	-0.28843	0.73859

Appendix 2. Simulation data quoted in literature 2

No.	x	y	z	No.	x	y	z
1	6.594000	0.000000	11.421143	14	-11.058732	8.034639	-7.892000
2	12.740966	0.000000	7.356000	15	-6.243993	4.536527	-13.367968
3	15.196000	0.000000	0.000000	16	-6.627467	-4.815137	14.188960
4	14.535370	0.000000	-8.392000	17	-11.571592	-8.407254	8.258000
5	7.258000	0.000000	-12.571224	18	-11.462152	-8.327742	0.000000
6	2.374487	7.307918	13.309078	19	-10.758862	-7.816772	7.678000
7	4.096674	12.608267	7.654000	20	-6.400942	-4.650557	-13.703986
8	4.074080	12.538729	0.000000	21	2.539502	-7.815783	14.233994
9	3.751984	11.547420	-7.010000	22	4.275442	-13.158458	7.988000
10	2.411569	7.422045	-13.516924	23	4.552438	-14.010964	0.000000
11	-6.781180	4.926816	14.518049	24	4.478831	-13.784424	-8.368000
12	-10.122691	7.354566	7.224000	25	2.443088	-7.519053	-13.693593
13	-12.442681	9.040137	0.000000				

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基于遗传算法的球度误差评定

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摘要: 首先对球度公差评定问题进行了综述。然后根据圆度公差的数学定义, 引申提出球度公差最小区域条件下的评定模型, 并给出遗传算法的适应度函数。随后给出算法实现中的关键问题。最后用实例对算法进行了检验, 计算结果表明基于遗传算法的球度误差优化算法不仅符合最小区域条件, 而且易于理解和实现, 能够获得全局最优解, 保证了高精度、高效率。

关　键　词: 球度误差; 模糊推理; 遗传算法

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收稿日期: 2002-01-14; 修订日期: 2002-05-20
基金项目: 国家自然科学基金资助(No. 59975025)