第 11 卷 第 2 期
2003 年 4 月

光学　精密工程
Optics and Precision Engineering

Vol. 11 No. 2
Apr. 2003

**Article ID** 1004-924X(2003)02-0181-07

# Novel method for solving maximum inscribed circle

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**Abstract :** A novel method has been proposed according to the minimum criterion for solving maximum inscribed circle. Two mathematical formulae have been developed for the establishment of the center of the maximum inscribed circle for the profile measured in such a way that a circle is decided by putting the coordinate values of the three points selected from the profile measured into the formulae developed. These three points form an acute triangle. The formulae can be used two or three times. Another point selected from the profile replaces one of the former three points each time. According with the minimum criterion , the final circle is just the maximum inscribed circle , of the profile measured when the whole profile measured is outside the circle. A flow chart of program and two examples are given in the paper. There is no principle error or method error in the results calculated by the formulae.

**Key words :** roundness error ; maximum inscribed circle (MIC) ; roundness measurement
**CLC number :** TB92 **Document :** A

## 1 Introduction

To evaluate precisely the roundness error of mechanical parts according with MIC , it is key to decide the MIC(s center of the profile measured , but it is difficult to do so because there is no mathematical formula available now. To get the desired results for roundness error evaluating , data obtained from measurement devices must be analyzed using appreciate computer - based an algorithm , and this algorithm must follow the specifications laid down in the Standards. Moreover , the algorithm must be efficient , reliable in evaluating roundness error. In the past decade , many methods have been developed to obtain the maximum inscribed circle solution of roundness error.

Liu Shugui , et al. [1] proposed a approach to obtain the maximum inscribed circle by calculational geometry method. Chetwynd[2] adopted limacon approximation for the circle and formulated the circularity problem as a linear programming problem. Samuel[3] used computational geometric techniques to evaluate roundness. Shunmugan[4] has suggested a new simple approach called the median technique , which gives minimum value of circularity error. Cui Changcai , et al. [5] employed a genetic algorithm in roundness error evaluation. Bourdet and Clement[6] used a small displacement screw model to get an approximate solution based on the minimax criterion. Murthy[7] proposed a two - dimensional (2 - D) simplex search method. The search is easy to code and efficient. Timothy Weber , et al. [8] employ a unified linear approximation technique for roundness error evaluation. Lai and Wang[9] proposed a computational geometric technique to solve the roundness problem , but their solution falls short of finding the minimum radial separation. Le and Lee[10] also described a similar approach. Roy and Zhang[11] proposed a solution based on the similar technique. Jyunping Huang[12].

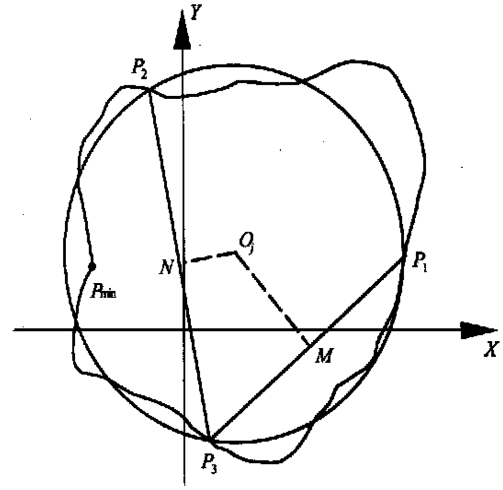
**Received date :** 2003-01-17 ; **修订日期 :** 2003-03-24.

proposed an exact solution using Voronoi diagrams. Tsukada et al.[13] used the Neldermead simplex method to evaluate the circularity. Wen - Yuh Jywe et al.[14] constructed three mathematical models to solve the min - max problem for evaluating roundness. Jyunping Huang[15] proposed a new strategy for improving the computational efficiency of evaluating roundness.

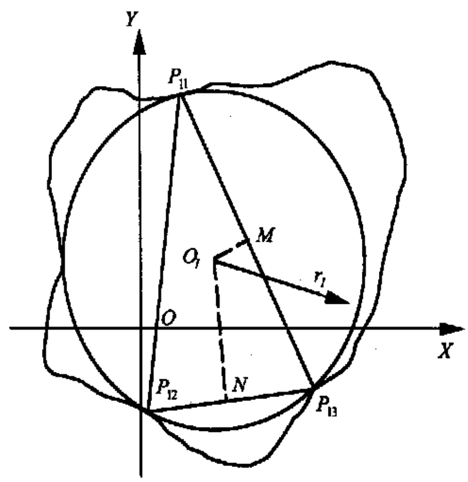
In this paper, a novel method —intersectant chords method, is proposed. The mathematical formulae, educed from the method, for the precise solution of maximum inscribed circle are given. The MIC of the profile measured can be easily calculated by using the mathematical formulae two or three times. The excellence of the method proposed here is that there is no principle error or method in the results calculated by the formulae, differing from any other method mentioned above.

## 2 Principle

The basic principle, deciding the center of maximum inscribed circle by the method presented in this paper, can be described as follows: First, draw a circle. The center of the circle is the intersection point of the vertical bisectors of two intersectant chords, which have same one end-point, on measured profile, for example $P_1P_3$ and $P_2P_3$ are two intersectant chords, $O_jM$ is the vertical bisector of $P_1P_3$, $O_jN$ is that of $P_2P_3$, and $O_j$ is the intersection point of $O_jM$ and $O_jN$ in Fig. 1(a). The radius of the circle is the distance from the center to these three points respectively, and these three points form an acute triangle. Second, check to see whether the whole measured profile is outside absolutely the circle or not. Finally, replace one end-point of these two intersectant chords by point Pmin if it is not and start next calculate cycle until the whole profile measured is outside the circle. It is necessary that the three end-points of the intersectant chords must form an acute triangle at all time, for example point Pmin replaces P3 in Fig. 1 (a). The final circle is just the maximum inscribed circle of the profile measured when the whole profile measured is outside the circle (shown in Fig. 1 (b)).




(a) A primal reference circle




(b) The MIC of the profile measured

Fig. 1 Principle of method proposed

## 3 Deciding the center of maximum inscribed circle

### 3.1 Mathematical model

As for roundness error evaluation, except the method of least-square circle, there is no formula available now to calculate directly the center of reference circle. So, first of all, the mathematical model, from which the formulae to calculate the center of reference circle can be deduced, should be built up.

In Fig. 1, it is supposed that point $O$ and $O_I$ is respectively the center of measurement system and maximum inscribed circle, $P_{I1}$, $P_{I2}$ and $P_{I3}$ are the three contact points at which the measured profile contacts with the maximum inscribed circle. Then the mathematical models to decide the center of maximum inscribed circle as well as to evaluate roundness error can be built up as follows:

$$O_I(X_I, Y_I) = F_I(P_{I1}, P_{I2}, P_{I3}), \tag{1}$$

$$f_I = \max_{I \le i \le n}\{r_{Ii}\} - r_I, \tag{2}$$

$$r_I = \sqrt{(X_{I1} - X_i)^2 + (Y_{I1} - Y_I)^2}, \tag{3}$$

$$r_{Ii} = \sqrt{(X_i - X_I)^2 + (Y_i - Y_I)^2}, \tag{4}$$

### 3.2 Formulae

Hereinto,

$n$ - the maximum number of the points measured on profile;

$r_{Ii}$ - the distance from the point $P_i$ on the profile measured to point $O_I$ ($i = 1, 2 \ldots, n$);

$f_I$ - the roundness error value evaluated according with maximum inscribed circle.

In Fig. 1, it is supposed that the coordinate values of point $P_i$ ($i = 1, 2, 3$), $M$, $N$ and $O_j$ are respectively $X_i$ and $Y_i$, $X_M$ and $Y_M$, $X_N$ and $Y_N$, $X_j$ and $Y_j$. Then the formulae, which are used to decide the center of maximum inscribed circle by intersectant chords method, can be deduced as follows:

$$X_j = \frac{(Y_M - Y_N)(Y_3 - Y_1)(Y_3 - Y_2) + X_M(X_3 - X_1)(Y_3 - Y_2) - X_N(X_3 - X_2)(Y_3 - Y_1)}{(X_3 - X_1)(Y_3 - Y_2) - (X_3 - X_2)(Y_3 - Y_1)}, \tag{5}$$

$$Y_j = \frac{(X_N - M_M)(X_3 - X_1)(X_3 - X_2) - Y_M(Y_3 - Y_1)(X_3 - X_2) + Y_N(Y_3 - Y_2)(X_3 - X_1)}{(X_3 - X_1)(Y_3 - Y_2) - (X_3 - X_2)(Y_3 - Y_1)}. \tag{6}$$

### 3.3 Steps

The process to obtain the center of maximum inscribed circle can be divided into four steps. First, select three points from the profile measured in that way in Fig. 1 to obtain two intersectant chords. Second, calculate the coordinate values, $X_j$ and $Y_j$, of the intersection point $O_j$ of the vertical bisectors of these two intersectant chords as well as radius sequence $\{r_{ji}\}$ ($i = 1, 2, \cdots, n$) by using formula (5), (6) and (4), taking $X_j$ and $Y_j$ as the center of the circle. Third, check to see whether the whole profile measured is outside the circle. Finally, one end-point of these two intersectant chords is replaced by point $P_{\min}$ if it is not, and next calculation repetition is started. It is necessary that the three end-points of the intersectant chords must form an acute triangle at all time, for example point P3 is replaced by point Pmin in Fig. 1 (a). The overlap segment from second step to final step is run repeatedly until the whole profile measured is outside the circle. The circle obtained finally is just the maximum inscribed circle of the profile measured. The flow chart to calculate the maximum inscribed circle is given in Fig. 2.

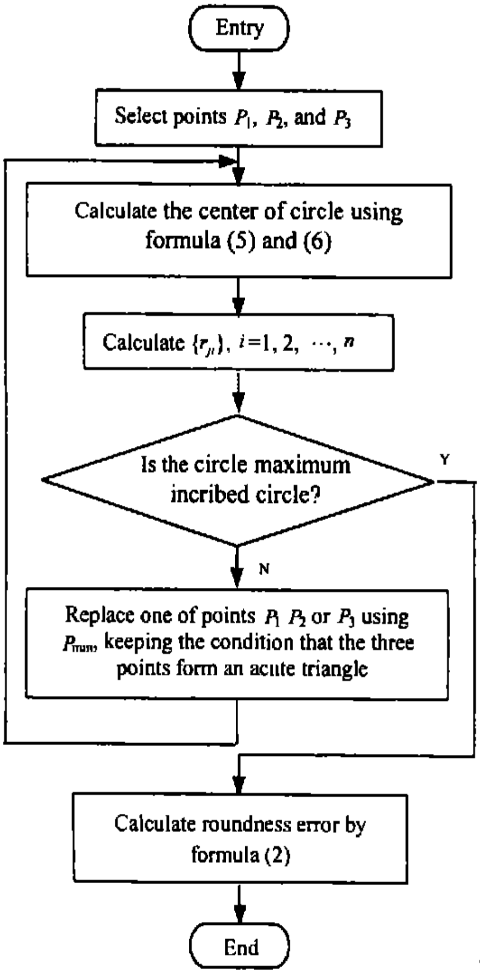
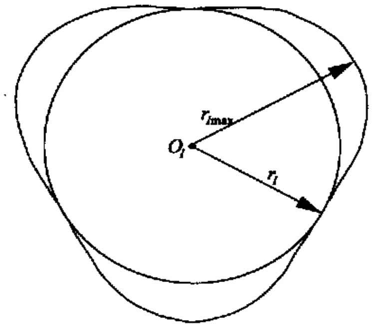
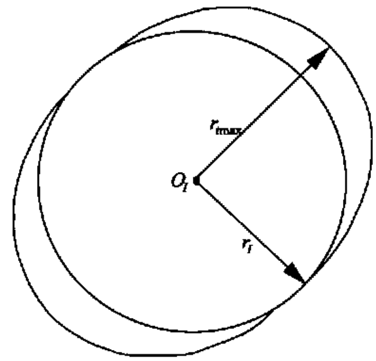



Fig. 2 Flow chart for calculation for maximum inscribed circle




(a) Profle 1




(b) Profile 2

Fig. 3 Maximum inscribed circle established by algorithm proposed for two practical round profiles

## 4 Examples and results

The novel method has been tested with the data of two practical profiles available in the appendix in this paper. The C language program, based on the algorithm presented here, finds accurately the maximum inscribed circle of the profiles. The results calculated are showed in Fig. 3 and Tab. 1.

Tab. 1 Results

| Profile 's number | | Profile 1 | Profile 2 |
|---|---|---|---|
| The center coordinate values of maximum inscribed circle | $X_I$ | 0.000 | -1.661 |
| | $Y_I$ | -0.039 | -1.661 |
| $r_{I\max}$ | | 111.961 | 114.349 |
| $r_I$ | | 88.039 | 88.031 |
| Roundness error $f_I$ | | 23.922 | 26.318 |

## 5 Conclusion

In this paper, the mathematical models depending on the method proposed here are described. Also, considering the need of computer programmers, the algorithm to decide maximum inscribed circle is developed. Without the convergence problem and linearizing error in optimal methods, the algorithm given here provides a novel strategy for the precise solution of maximum inscribed circle. As for the selection of primal three points, it merely affects the time spent on running the program, but it has no relation with the final results. However, the effect is baby-size. The precision of the results is onlyrelated to the last bit of the variable used in program. The method proposed in the paper has been used in the apparatus for roundness error measurement, and its efficiency and reliability has been proven by the practical application.

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# 求解最大内切圆的一种新方法

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**摘要** :提出了一种求解最大内切圆的新方法 ,给出了最大内切圆圆心坐标值的计算公式 ,该方法的基本思路是 :首先在被测轮廓上选取初始三点 ,并保证这三点构成一个锐角三角形 ;接下来通过给出的公式计算出这三点所在圆的圆心坐标值 ,被测轮廓各点到该圆心的距离序列 ;最后判断该圆半径是否等于上述距离序列中的最小值 ,如果条件不满足 ,用最短距离所对应的被测轮廓点代替上述三点之一 ,并保证新的三点仍然形成一个锐角三角形 ,然后重复上述计算和判断过程 ,直至条件满足。最后一次计算所得到的圆心恰好是被测轮廓的最大内切圆圆心 ,该方法的优点在于不存在原理误差 ,速度快 ,一般二、三次计算即可 ,给出了程序流程图。

**关　键　词** :圆度误差评定 ;最大内切圆 ;锐角三角形
**中图分类号** : TB92　　**文献标识码** :A

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**Appendix**

Profile 1

| No. | $X_i$ | $Y_i$ |
|---|---|---|
| 1 | 0.000 | 100.000 |
| 2 | 10.143 | 102.985 |
| 3 | 20.810 | 104.617 |
| 4 | 31.721 | 104.571 |
| 5 | 42.511 | 102.631 |
| 6 | 52.769 | 98.724 |
| 7 | 62.096 | 92.933 |
| 8 | 70.153 | 85.482 |
| 9 | 76.711 | 76.711 |
| 10 | 81.674 | 67.028 |
| 11 | 85.093 | 56.858 |
| 12 | 87.155 | 46.585 |
| 13 | 88.145 | 36.511 |
| 14 | 88.409 | 26.819 |
| 15 | 88.293 | 17.562 |
| 16 | 88.090 | 8.676 |
| 17 | 88.000 | 0.000 |
| 18 | 88.090 | -8.676 |
| 19 | 88.293 | -17.562 |
| 20 | 88.409 | -26.819 |
| 21 | 88.145 | -36.511 |
| 22 | 87.155 | -46.585 |
| 23 | 85.094 | -56.858 |
| 24 | 81.674 | -67.028 |
| 25 | 76.711 | -76.711 |
| 26 | 70.153 | -85.482 |
| 27 | 62.096 | -92.933 |
| 28 | 52.769 | -98.724 |
| 29 | 42.511 | -102.631 |
| 30 | 31.721 | -104.571 |
| 31 | 20.810 | -104.617 |
| 32 | 10.143 | -102.985 |
| 33 | 0.000 | -100.000 |
| 34 | -9.460 | -96.052 |
| 35 | -18.208 | -91.540 |
| 36 | -26.336 | -86.817 |
| 37 | -34.026 | -82.145 |
| 38 | -41.510 | -77.660 |
| 39 | -49.018 | -73.361 |
| 40 | -56.726 | -69.120 |
| 41 | -64.711 | -64.711 |
| 42 | -72.928 | -59.851 |
| 43 | -81.200 | -54.256 |
| 44 | -89.229 | -47.694 |
| 45 | -96.631 | -40.026 |
| 46 | -102.979 | -31.238 |
| 47 | -107.864 | -21.456 |

Profile 2

| No. | Xi | YI |
|---|---|---|
| 1 | 0.000 | 100.000 |
| 2 | 10.031 | 101.848 |
| 3 | 20.405 | 102.582 |
| 4 | 30.964 | 102.074 |
| 5 | 41.516 | 100.227 |
| 6 | 51.843 | 96.992 |
| 7 | 61.716 | 92.365 |
| 8 | 70.906 | 86.399 |
| 9 | 79.196 | 79.196 |
| 10 | 86.399 | 70.906 |
| 11 | 92.365 | 61.716 |
| 12 | 96.992 | 51.843 |
| 13 | 100.227 | 41.516 |
| 14 | 102.074 | 30.964 |
| 15 | 102.582 | 20.405 |
| 16 | 101.848 | 10.031 |
| 17 | 100.000 | 0.000 |
| 18 | 97.189 | -9.572 |
| 19 | 93.575 | -18.613 |
| 20 | 89.314 | -27.093 |
| 21 | 84.549 | -35.021 |
| 22 | 79.393 | -42.436 |
| 23 | 73.929 | -49.398 |
| 24 | 68.203 | -55.973 |
| 25 | 62.225 | -62.225 |
| 26 | 55.973 | -68.203 |
| 27 | 49.398 | -73.929 |
| 28 | 42.436 | -79.393 |
| 29 | 35.021 | -84.549 |
| 30 | 27.093 | -89.314 |
| 31 | 18.613 | -93.575 |
| 32 | 9.572 | -97.189 |
| 33 | 0.000 | -100.000 |
| 34 | -10.031 | -101.848 |
| 35 | -20.405 | -102.582 |
| 36 | -30.964 | -102.074 |
| 37 | -41.516 | -100.227 |
| 38 | -51.843 | -96.992 |
| 39 | -61.716 | -92.365 |
| 40 | -70.906 | -86.399 |
| 41 | -79.196 | -79.196 |
| 42 | -86.399 | -70.906 |
| 43 | -92.365 | -61.716 |
| 44 | -96.992 | -51.843 |
| 45 | -100.227 | -41.516 |
| 46 | -102.074 | -30.964 |
| 47 | -102.582 | -20.405 |

| | | |
|---|---|---|
| 48 | -110.946 | -10.927 |
| 49 | -112.000 | -0.000 |
| 50 | -110.946 | 10.927 |
| 51 | -107.864 | 21.456 |
| 52 | -102.979 | 31.238 |
| 53 | -96.631 | 40.026 |
| 54 | -89.229 | 47.694 |
| 55 | -81.200 | 54.256 |
| 56 | -72.928 | 59.851 |
| 57 | -64.711 | 64.711 |
| 58 | -56.726 | 69.120 |
| 59 | -49.018 | 73.361 |
| 60 | -41.510 | 77.660 |
| 61 | -34.026 | 82.145 |
| 62 | -26.336 | 86.817 |
| 63 | -18.208 | 91.540 |
| 64 | -9.460 | 96.052 |

| | | |
|---|---|---|
| 48 | -101.848 | -10.031 |
| 49 | -100.000 | -0.000 |
| 50 | -97.189 | 9.572 |
| 51 | -93.575 | 18.613 |
| 52 | -89.314 | 27.093 |
| 53 | -84.549 | 35.021 |
| 54 | -79.393 | 42.436 |
| 55 | -73.929 | 49.398 |
| 56 | -68.203 | 55.973 |
| 57 | -62.225 | 62.225 |
| 58 | -55.973 | 68.203 |
| 59 | -49.398 | 73.929 |
| 60 | -42.436 | 79.393 |
| 61 | -35.021 | 84.549 |
| 62 | -27.093 | 89.314 |
| 63 | -18.613 | 93.575 |
| 64 | -9.572 | 97.189 |