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Comparison of genetic algorithm based evaluation of roundness with evaluation of roundness based on least squared method

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Abstract: The evaluation of roundness based on genetic algorithm method (GAM) is compared with the evaluation of roundness based on least square method (LSM) with their advantages and drawbacks discussed in detail using the model proposed, which features bounds control data to simulated the maximum inscribed and maximum circumscribed circles defined under minimum zone conditions, and randonly produced data to simulate the randomness and uncertainties of test points under actual conditions. The computational results show that the accuracy of GAM is better than that of LSM.

Key words: genetic algorithm-based method (GAM); least squared method (LSM); circularity
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1 Introduction

With the development of computer and automation technology, the precision methods for form error computation are available. Because the methods must be based on proved mathematical principles and tolerance specifications, and must be efficient and accurate[1-2], it is difficult but indispensable to research better algorithms for dimension and tolerance evaluation.

The least squared method (LSM) is relatively mature at present, which has been broadly utilized and generally used as the initial value of other methods[3-4]. LSM is on the basis of sound mathematical principles that minimize the sum of the squared deviations of the measured points from the nominal feature. It does not follow the standards intently and will not guarantee the minimum zone solution specified in the tolerance standard. The deviation values and geometric error determined by LSM will be generally larger than the actual ones, which may lead to rejection of good parts.

Therefore, it is so necessary to develop minimum zone based algorithms that have been proposed by now such as computation geometry based evaluation algorithm[5], convex-based mathematical method[6], approach to form tolerance evaluation using non-uniform rational B-splines[4] and function-oriented form inspection[1,7] and so on.

The genetic algorithm based method (GAM) has been studied[8]. The genetic algorithm based data optimization has overcome some of the exiting

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drawbacks to some extent. The genetic algorithm is an adaptive and probabilistic search algorithm with an iterative procedure that simulates the evolution process of the nature. It begins with an initial population of solutions which can be given by other simple and approximate methods or experiment values, and by some genetic strategy the initial seeds produce new individuals of next generation by such operations as reproduction, crossover and mutation. In the process of production the best individuals are reserved, but the worst ones are rejected. Through some generations of iterations the desired solution can be obtained. The optimization based on this technology has been improved and utilized in many fields[9,10].

This paper describes a wheel-form model to confirm the fact that the GAM is better than the traditional methods such as LSM. In the paper a comparison model is firstly constructed, then the example results are analyzed in detail, and finally a conclusion is given.

2 Mathematical model of comparison

The hypothetical circular model of comparison is illustrated in Fig. 1. Firstly, produce two control circles are, as in shown Fig. 1, which are taken as the maximum inscribed circle and minimum circumscribed circle just as the minimum zone definition of the circularity specifies. At the same time determine some controls data on the circles that are distributed uniformly and alternatively between the inner and outer circles to assure that they satisfy the minimum zone condition. Their radii are marked as r_{max} and r_{min}, respectively. According to the specifications of circularity evaluation, the fitted radius r of the data is $(r_{max}+r_{min})/2$, and the circularity error f is $(r_{max}-r_{min})$. Secondly, randomly produce some random data lying in the area bounded by the concentric control circles to simulate the real world of measurement. And then change the numbers of data to prove which algorithm will be more accurate and stable.

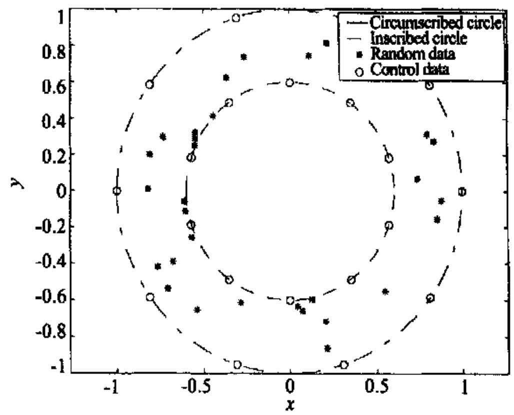

Fig. 1　Mathematical model of comparison

2.1 Least squared method

According to the principle of LSM, the mathematical expression can be given as:

$$f=\sum_{j=1}^{n}(r_j-r_d)^2, \quad (1)$$

where r_j is the distance between the j th measurement data and the center, r_d is the nominal radius of the fitted circle. Let $(x_j, y_j)(j=1,2,\cdots,n)$ denote simulation points produced by the method above for convenience, then Equ. (1) can be given as:

$$f=\sum_{j=1}^{n}\left(\sqrt{(x_j-a)^2+(y_j-b)^2}-r_d\right)^2, \quad (2)$$

where (a, b) is the center of the fitted circle.

Therefore the evaluation of the form error of a circle by least squares method is to minimize the value of function f, namely minimize (f) and at the same time obtain the parameters of the fitted feature.

2.2 Genetic algorithm based method

This method simulates the strategy of "survival of the fittest" of natural evolution and obtains the superior individuals from the population. As stated in the introduction subsection, the algorithm starts with an initial value which is given by the least squares method here, that is the center (a, b). And they are taken as the optimization variables, represented as $(a+\Delta a, b+\Delta b)$. In the population of the evolution, the individuals (called chromosomes) can be given as $(\Delta a, \Delta b)$ if the real number is selected to represent every bit of chromosome (called gene). The chromosome moves from generation to generation until the termination condition is met. If the ith chromosome of the Tth generation is given as $(\Delta ai(T), \Delta bi(T))$, then the radius of the fitted circle can be given as[11-12]:

$$r_{ji} = \sqrt{\{x_j - [a + \Delta a(T)]\}^2 + \{y_j - [b + \Delta b(T)]\}^2} = r(x_j, y_j, \Delta a_i, \Delta b_i), \quad (3)$$

And the objective function of the algorithm can be expressed as:

$$obj(x_j, y_j, \Delta a_i, \Delta b_i) = r_{max}(x_j, y_j, \Delta a_i, \Delta b_i) - r_{min}(x_j, y_j, \Delta a_i, \Delta b_i), \quad (4)$$

So the fitness function of genetic algorithm is denoted as:

$$fit(\Delta a_i, \Delta b_i) = [obj(x_j, y_j, \Delta a_i, \Delta b_i)]^{-1}, \quad (5)$$

Therefore the genetic algorithm evolves through operations such as selection, crossover and mutation, etc. by the fitness value in terms of Equ. (5). The greater the fitness, the stronger the individual. The best individual can be obtained in the end.

3 Computation examples and results analysis

The idea is verified by the example experiments. The radii of the control circles are assigned with 1.0050 mm and 0.9950 mm, respectively and the center of concentric circles is located at the origin (0, 0) of the coordinate system. The number of the control data is 10 distributed uniformly on each circle and the interval is 36°. The angle between the adjacent data on the inner and outer circles is 18° just like one of them is rotated 18° relative to the other around the center. In the experiment 6 groups of random data have been produced and together with the control data mentioned above they are used to simulate the real cases of different number of sampling data. The produced data are given in the appendix 1 and 2. The results are shown in table 1, where a and b are the center coordinates of the fitted circle, r is the fitted radius and f is the circularity error. From the minimum zone condition and the produced data, the theoretical ideal solution is also given in table 1.

Tab. 1 Computation results of the two methods

Method		Control data	Random data	(a, b)	r	r_{max}	r_{min}	f (μm)
Ideal value		20		(0.0000, 0.0000)	1.0000	1.0050	0.9950	10.0000
LSM	G1	20	20	(0.0003, 0.0010)	1.00002	1.00604	0.9940	12.0442
	G2	20	30	(-0.0002, -0.0008)	1.00001	1.00582	0.9942	11.6231
	G3	20	40	(0.0005, 0.0000)	1.00002	1.0055	0.994532	10.968
	G4	20	50	(-0.0004, -0.0001)	0.999998	1.0054	0.994596	10.8038
	G5	20	50	(-0.0002, 0.0000)	1.00001	1.0052	0.994817	10.3827
	G6	20	50	(0.0003, 0.0001)	1.0000	1.00532	0.994691	10.6267
GAM	G1	20	20	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919
	G2	20	30	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919
	G3	20	40	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919
	G4	20	50	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919
	G5	20	50	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919
	G6	20	50	(-0.000093, -0.000101)	1.00001	1.00515	0.994859	10.2919

Note: G denotes the group of the simulation data.

The results of the LSM show that the circularity errors of different groups are all greater than the ideal value but they decrease as the random data increase. In other words it is essential to sample enough data for a high accuracy although it is difficult to know the definite numbers of data. Obviously some good parts evaluated by this method will inevitably be rejected.

The GAM is operated under the following conditions: population size is 30, real number coding is adopted, crossover probability is 0.35, mutation probability is 0.06 and the termination condition is the maximum generation 100. The results obtained by the genetic algorithm-based method should not be affected by the number and position of random data in theory, which has been proved by the experiment that they have nothing to do with the number and position almost in fact under considerations of accuracy of the algorithm implementation itself and the memory capacity and computation ability of the computer.

The four-six groups with equal random data number have validated the stability of the same algorithm. The comparison results indicate that the genetic algorithm can obtain a more accurate and stable solution. The circularity curves of the two methods are shown in Fig. 2. The error of sampling data in group 1 relative to the fitted circle is illustrated in Fig. 3 and 4, which show that the results given by the genetic algorithm-based method appear in a narrower zone.

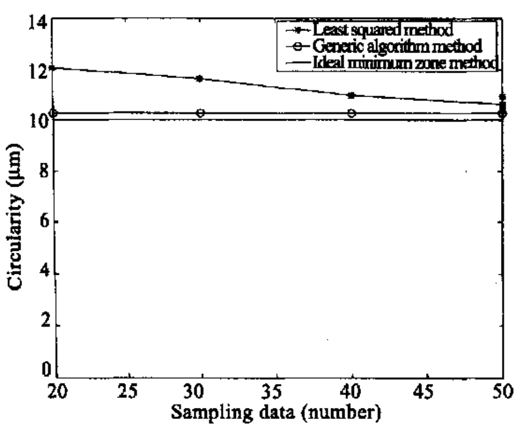

Fig. 2　Comparison of circularity obtained with two methods

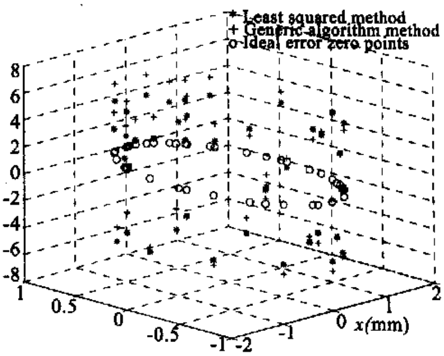

Fig. 3　Relative error distribution of sampling data in group 1

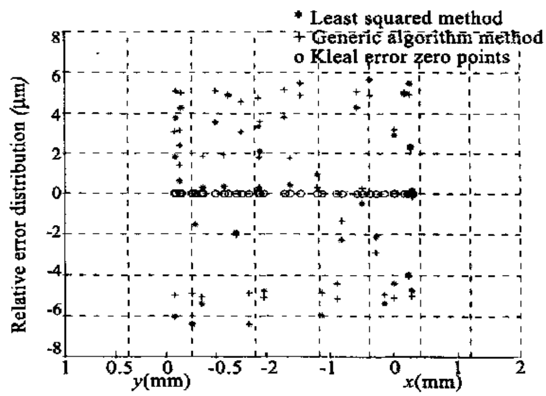

Fig. 4　Side view of the relative error of sampling data in group 1

4　Conclusion

The computation results obtained with the comparison model have verified that the genetic algorithm has better performance than the least squared method. So the former is suitable for requirements with high accuracy and efficiency in precision industry. The latter can be used when the accuracy is not very strict or the solution is used as the initial value of other methods. There is no doubt that the genetic optimization's application in the form and tolerance inspection is promising with automatic, intelligent and prevailing network technologies.

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基于遗传算法的圆度公差评定法与采用最小二乘法评定的比较

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摘要: 根据提出的计算模型, 对基于遗传算法的圆度误差评定和传统上采用最小二乘法的评定算法进行了比较分析, 根据方法本身的特点和计算结果, 分析了二者的不同点以及在工程应用中的适用场合。所构造的模型包括边界控制点和区域随机点, 其中边界控制点模拟了由圆度误差最小区域条件所定义的最大内切圆和最小外切圆, 而区域随机点模拟了实际情况下测试点的随机性和不确定性。计算结果表明基于遗传算法的圆度评定法精度较高, 优于基于最小二乘法的评定算法。

关 键 词: 遗传算法; 最小二乘法; 圆度

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Appendix 1 The control data on the minimum circumscribed and maximum inscribed circles

On the minimum circumscribed circle			On the maximum inscribed circle		
No.	x	y	No.	x	y
1	0.8131	0.5907	11	0.9463	0.3075
2	0.3106	0.9558	12	0.5848	0.8050
3	-0.3106	0.9558	13	0.0000	0.9950
4	-0.8131	0.5907	14	-0.5848	0.8050
5	-1.0050	0.0000	15	-0.9463	0.3075
6	-0.8131	-0.5907	16	-0.9463	-0.3075
7	-0.3106	-0.9558	17	-0.5848	-0.8050
8	0.3106	-0.9558	18	-0.0000	-0.9950
9	0.8131	-0.5907	19	0.5848	-0.8050
10	1.0050	-0.0000	20	0.9463	-0.3075

Appendix 2 The random data between the minimum circumscribed and maximum inscribed circles

Group one: 20 random data					
No.	x	y	No.	x	y
1	-0.7180	-0.6967	11	-0.9956	-0.0706
2	0.6750	0.7425	12	0.8358	0.5520
3	-0.3694	0.9325	13	0.9893	0.1357
4	0.6634	0.7505	14	0.5407	0.8465
5	0.1671	0.9877	15	0.5595	-0.8287
6	-0.7986	0.6071	16	0.7224	-0.6856
7	0.6637	0.7542	17	0.3863	0.9243
8	0.0561	0.9983	18	0.9987	-0.0546
9	0.9009	-0.4412	19	-0.2357	-0.9690
10	-0.7760	0.6330	20	0.6923	-0.7247

Note ①: The method used to produce the data randomly is described as following equations:

$$x = r\cos\theta$$

$$y = r\sin\theta$$

where: r = a random data between the radii of the minimum circumscribed and maximum inscribed circles.

θ = a random angle in the range of $(0, 2\pi)$.

Note ②: Group 2– 4 of random data is produced by the method as the group 1 used except the different number. Although group 5 and 6 has the same number of data as group 4, they are still different because of the randomness of the r and θ.