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## Precise computation of planar straightness error using genetic algorithm

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**Abstract:** The computation model established for the problem is described, and the key techniques of the GA-based method including representation of chromosomes, generation of initial population, fitness function, and determination of genetic operators (selection, crossover and mutation, etc.) and stopping criteria for genetic operations, are discussed in detail. This algorithm was substantiated with typical examples and the results show that the GA-based method can be used to achieve precise computation of planar straightness errors.

**Key words:** straightness; error; genetic algorithm

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## 基于遗传算法的平面直线度误差的精确计算

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**摘要:** 提出一种基于遗传算法的平面直线度计算方法, 该方法计算原理简单, 借助于计算机技术, 可以较容易地实现计算精度要求较高时的直线度误差计算。文章首先给出问题的计算模型。接着介绍了在遗传算法实现中的关键技术, 包括染色体个体的表达、初始群体的产生方式、适应度函数、遗传算子(选择、交叉和变异等)的确定、遗传运算的停止准则等。最后应用实例对算法进行了验证计算和分析, 结果表明, 所设计的基于遗传算法的平面度误差计算方法可以实现平面直线度的精确计算。

**关键词:** 直线度; 误差; 遗传算法

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### 1 Introduction

It is an important research topic in mechanical industry to find out more precise methods for form error evaluation. They have an effect on the manufacturing precision so that they have been paid attention to for a long time by the scholars and engineers<sup>[1-3]</sup>. Recently, two kinds of methods are available for the form error verification. One is the least squared method that is based on the principle of minimizing the sum of the residual error between the

measurement and nominal values. Therefore it is an approximate method and usually used to situations with lower precision requirement or as initial computation values of other methods. Another kind of method is the minimum zone-based method. Because it conforms to the standard, it can give a precise solution. But no existing computation models are perfect up to now. On the basis of modern optimization computation methods—genetic algorithm (GA), a new model is devised in the paper which is fit for high precision measurement.

## 2 Computation model of planar straightness evaluation

According to the definition of planar straightness tolerance zone , it is an area bounded by two parallel straight lines. The distance between the two lines is the straightness tolerance<sup>[4]</sup>. The planar straightness error is the minimum distance between two parallel straight lines containing all the measurement points , as shown in Fig. 1.

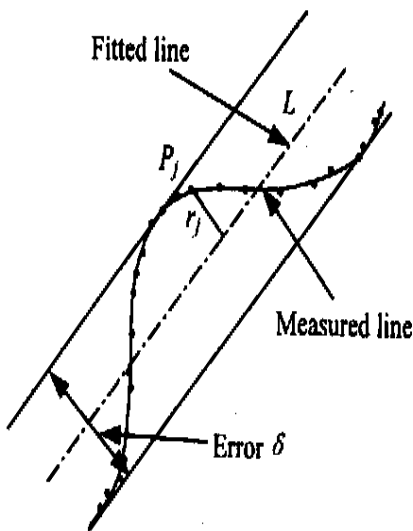


Fig. 1 Straight line error zone

Suppose that  $L$  is the fitted line with feature equation as  $y = ax + b$  , the directional distance between any measurement point  $P_j(x_j, y_j)$  ( $j = 1, 2, \dots, M$ ) and the line is given by :

$$r_j = \frac{y_j - ax_j - b}{\sqrt{1 + a^2}} \quad (1)$$

where ,  $a$  and  $b$  are the variables to be searched. The evaluation problem is to find out a desired line from a group of lines  $L_i(a_i, b_i)$  ( $i = 1, 2, \dots, N$ ). The variables are expressed as the form of an initial value plus a supplement , namely  $L_i(a_0 + a_i, b_0 + b_i)$  , then Equ. (1) has another form :

$$r_{ji} = \frac{y_j - (a_0 + a_i)x_j - (b_0 + b_i)}{\sqrt{1 + (a_0 + a_i)^2}} \quad (2)$$

Therefore , the objective function of the problem is defined as :

$$h(a_i, b_i) = \min_i \max_j r_{ji}(a_i, b_i, x_j, y_j) - \min_j r_{ji}(a_i, b_i, x_j, y_j) \quad (3)$$

## 3 Implementation principles and techniques of the GA-based method

The implementation techniques include many aspects , such as chromosome representation , initial population generation , fitness function expression , genetic operators(selection , crossover and mutation , etc.) determination , stopping criteria decision. The following subsections will describe their implementation in detail<sup>[5-6]</sup>.

### (1) Chromosome individual representation

The representation of the chromosomes is the first issue of the genetic computation. It determines how to represent the variables of the problem into chromosomes of the genetics. In the basic GA , the binary encoding is adopted , which has its strengths and limitations. The disadvantages include lower transformation precision and computation efficiency and it is not straight to understand and inconvenient to implement self-adaptable genetic operations. However the real float encoding utilizes the variables of the problem as the genes of the individuals , which does need the transformation from float and binary systems. Therefore there is no transformation precision loss and time consuming. Moreover the variables correspond to the genes of individual , which is convenient for understanding the action strength of the variables to the minimum objective. It is easy to adjust the genetic operators so as to improve the performance of the population. The individual or chromosome is given as :

$$X_i = ( a_i, b_i ) \quad (4)$$

where  $i$  is the serial number of the chromosome which corresponds to a candidate straight line. So  $i = 1, 2, \dots, N$ .

The  $N$  individuals constitute the initial evolution population whose values are randomly generated around the least squared solution in terms of uniform distribution.

### (2) Fitness function determination

GA does not require the problem with strict mathematical property. It only needs the optimization objective

and the influencing factors and modes to solve the problem. The fitness function is the bridge between the specific problem and GA that is the measure of the individual performance. According to the objective function Equ. (3), the fitness function is defined as:

$$f(X_i(g)) = f(a_i(g), b_i(g)) = h(a_i(g), b_i(g))^{-1}, \quad (5)$$

where,  $g$  is the evolution generation. When the algorithm runs at generation  $G$ , the  $i$ th individual is the optimum chromosome with the biggest fitness value, and the precision is satisfactory, then the error of the line is calculated by:

$$= \max_{j,m} r_{ji_0}(a_{i_0}, b_{i_0}, x_j, y_j) - \min_{j,m} r_{ji_0}(a_{i_0}, b_{i_0}, x_j, y_j), \quad (6)$$

(3) Genetic operations ——selection, crossover and mutation

The selection or reproduction operator is the main evolutionary operator which simulates the Darwinian evolution idea to choose the better individuals with strong adaptable ability as the intermediate population of the genetic operation. The roulette wheel selection is adopted here. Firstly compute the fitness value of every individual of the population. Then obtain the selection probability of the individual according to the fitness.

$$p(X_i) = \frac{f(X_i)}{\sum_{k=1}^N f(X_k)}, \quad (7)$$

The selection probability is used to choose the individuals with bigger fitness as the new population of evolution.

The crossover or recombination operator is the main variation operator that simulates the Mendel genetic theory. In the process of evolution, the population must exchange and add new information to increase the survival and competition ability. Therefore the population can survive and improve. The crossover operation is to change some genes of the individual to obtain new individuals.

The single point crossover is used to exchange

two corresponding parts of two parent individuals and get a new individual with bigger fitness value. Let two parent individuals as  $X_k = (a_k, b_k)$  and  $X_l = (a_l, b_l) (k, l = N)$ , then two new individuals can be given by:

$$\begin{cases} X_k = (a_k, b_l) \\ X_l = (a_l, b_k) \end{cases}, \quad (8)$$

If two new individuals both are unsatisfactory, repeat the crossover operation until the desired one appears.

The mutation operator is an assistant operator that can enrich the genetic information. It is necessary for the genetic algorithm to fulfill the searching assignment. It can strengthen the local searching ability and avoid the premature status.

The single mutation adopted is to substitute a new value for the mutated gene and get an individual. Although the operation is simple it is beneficial to the evolutionary population.

(4) Stopping criteria of GA

The maximum number of iterations is looked as the termination condition that is usually obtained through many a genetic run.

### 4 Verification examples and results analysis

The examples in literature<sup>[7]</sup> are used to validate the proposed method. The method adopted

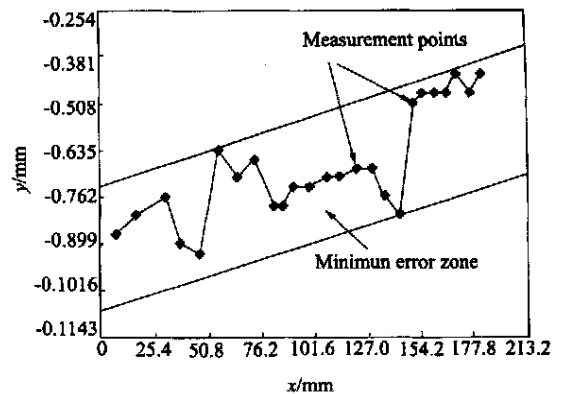


Fig.2 Distribution of the measurement points in example 1 and the minimum error zone

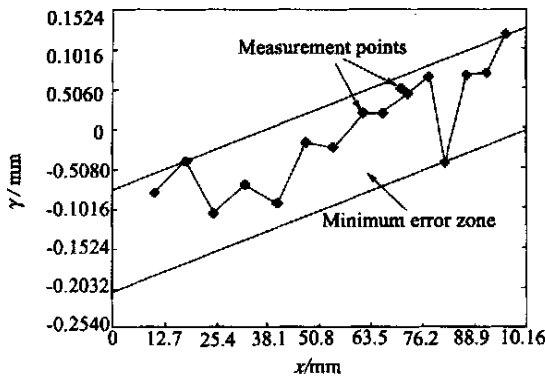


Fig.3 Distribution of the measurement points in example 2 and the minimum error zone

by literature<sup>[7]</sup> was based on convex hull. Figs. 2-3 show the distribution of the measurement points in plane and the minimum zone obtained by the GA-based method in the paper.

The computation conditions are given in Table 1 and 2<sup>[7]</sup>, and the results are shown in Tab. 3.

Tab.1 Parameters of genetic algorithm run

Example number	1	2
Population size	30	30
Crossover probability	0.85	0.85
Mutation probability	0.05	0.06
Maximum generations	2000	2000

Tab.2 Computation initial values and comparison results<sup>[7]</sup>

Example number	1	2
Initial value $a_0$ (mm)	0.005 656 326	0.053 162 454
Initial value $b_0$ (mm)	-0.091 147 90	-0.125 825 504
Least squared solution (mm)	0.037 165 28	0.136 573 006
Solutions based on convex hull (mm)	0.033 299 4	0.131 724 4

Tab.3 Computation results by GA-based method

Example number	Parameter $a$ (mm)	Parameter $b$ (mm)	Solutions based on GA (mm)
1	0.004 527 804	-0.088 897 612	0.033 309 814
2	0.051 229 514	-0.127 722 63	0.131 719 574

As shown in Tab.2 and 3, the improved GA-based method can correctly compute the planar straightness error. The method begins with the

least squared solution and can greatly increase the computation precision. The results are equal to or better than those by the convex hull-based method in literature<sup>[7]</sup> which is more complicated than the proposed method and not easy to computer implement because of its geometrical model.

The evolution process of the examples are illustrated in Figs. 4-5.

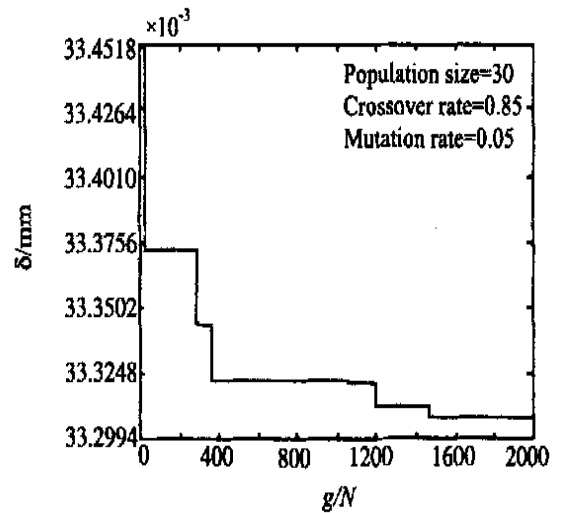


Fig.4 Evolution process of example 1

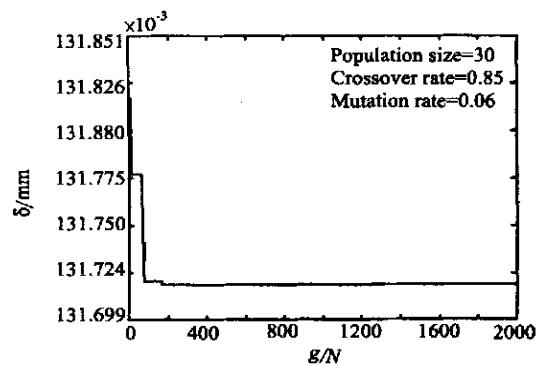


Fig.5 Evolution process of example 2

Analyzed from three Figs. 4-5 above, the whole population does evolve towards the minima and rapidly converges at the initial stage, basically at the beginning generations. Then the population goes to a more stable stage as the generations increase until the minimum solution appears.

## 5 Conclusions

In order to evaluate the planar straightness error, a GA-based method is proposed. It is on the basis of natural evolution principle "struggle for life

or survival of the fittest" to optimize the population of individual solutions and give the correct solution.

The examples prove that the method is simple in principle and higher in computation precision than the least squared method and higher or equal to the convex hull-based method.

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